CERN

U.S. Particle Accelerator School

Education in Beam Physics and Accelerator Technology

Collective effects in Beam Dynamics

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USPAS, one-week course, 19-24 January, 2015

http://uspas.fnal.gov/index.shtml

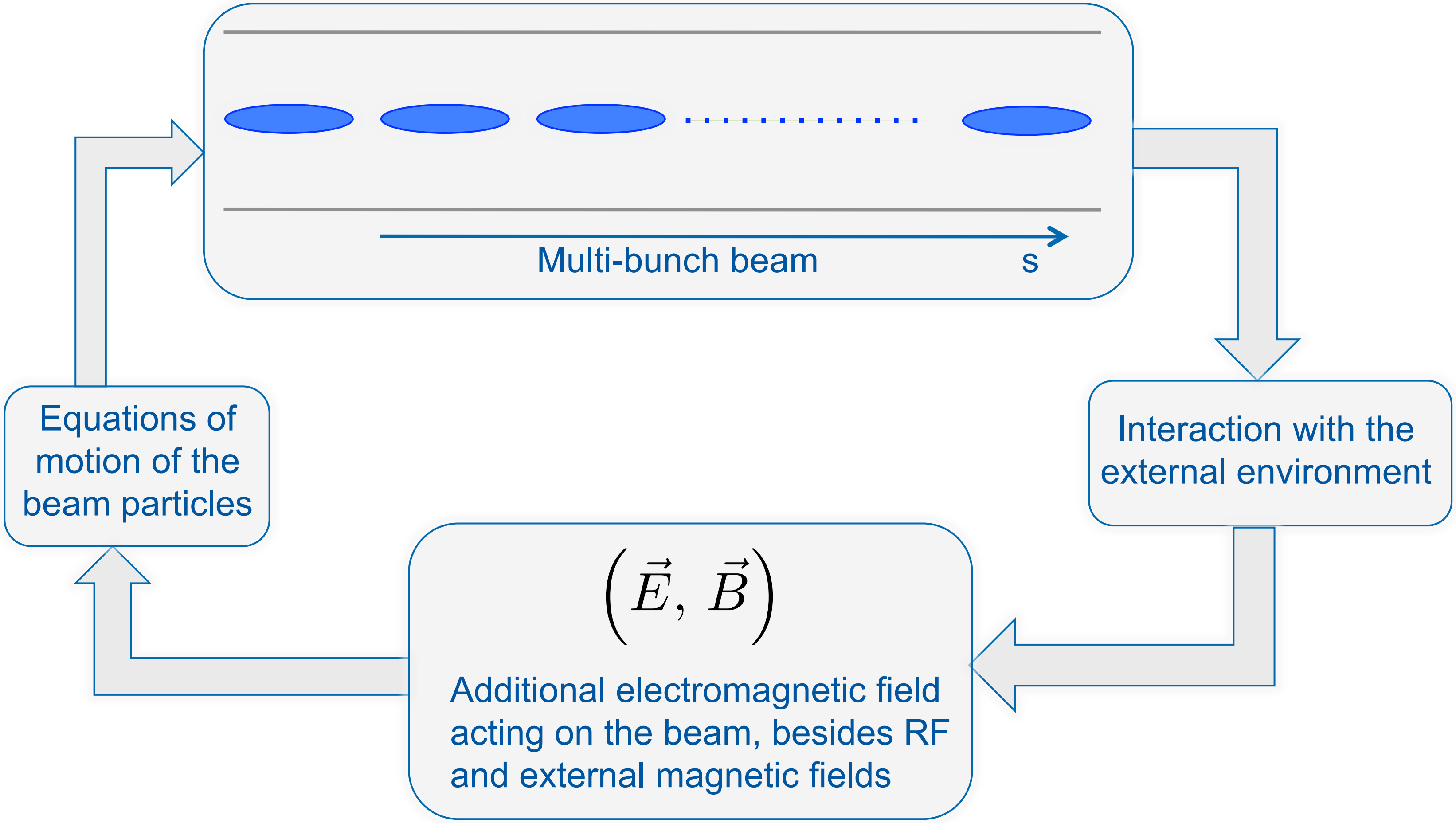
Outline

1. Introductory concepts
   - Collective effects
   - Transverse single particle dynamics including systems of many non-interacting particles
   - Longitudinal single particle dynamics including systems of many non-interacting particles
2. Space charge
   - Direct space charge (transverse)
   - Indirect space charge (transverse)
   - Longitudinal space charge

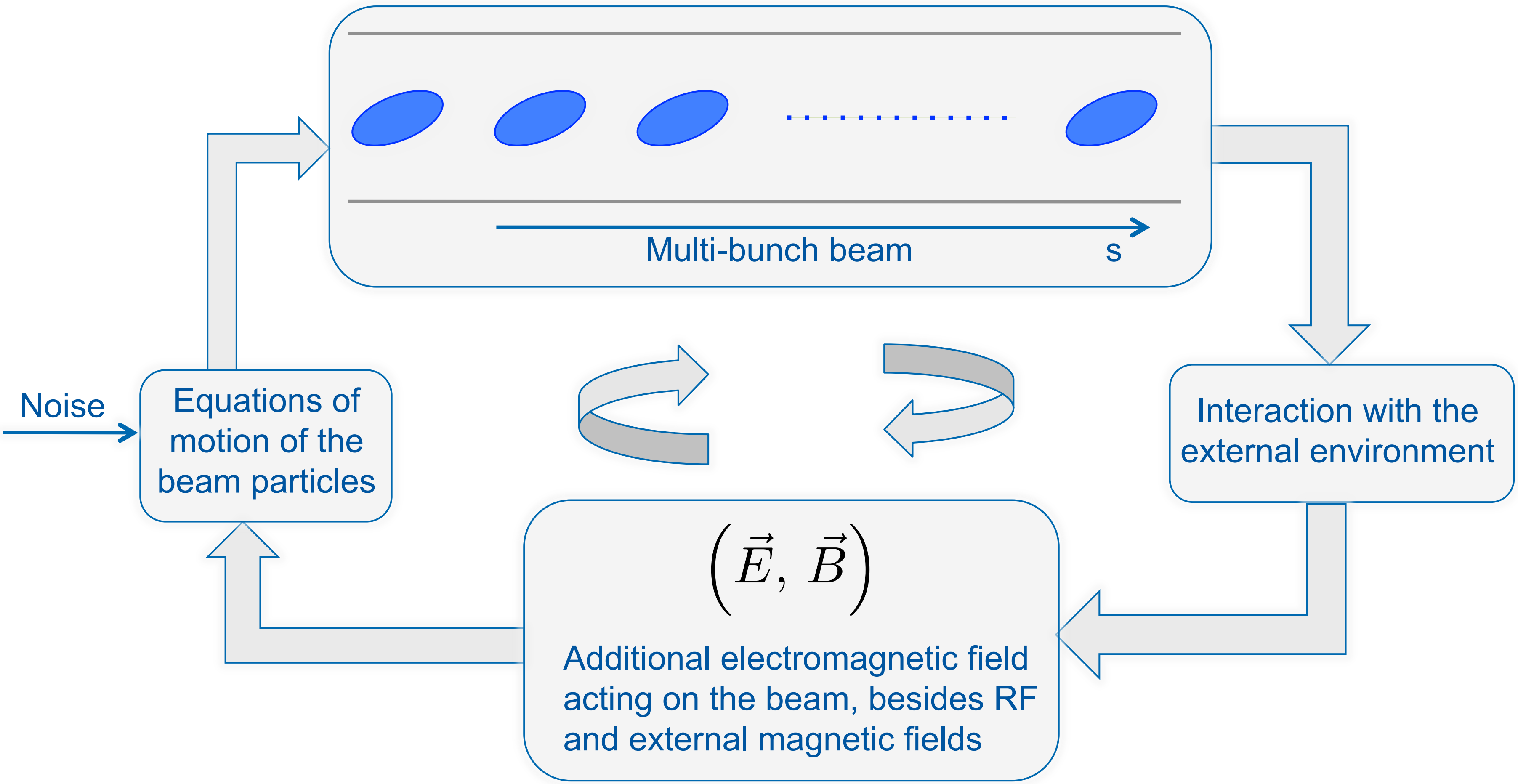
Outline

3. **Wake fields and impedance**
   - Wake fields and wake function
   - Definition of beam coupling impedance
   - Examples – resonators and resistive wall
   - Energy loss
   - Impedance model of a machine

Introduction to the general problem

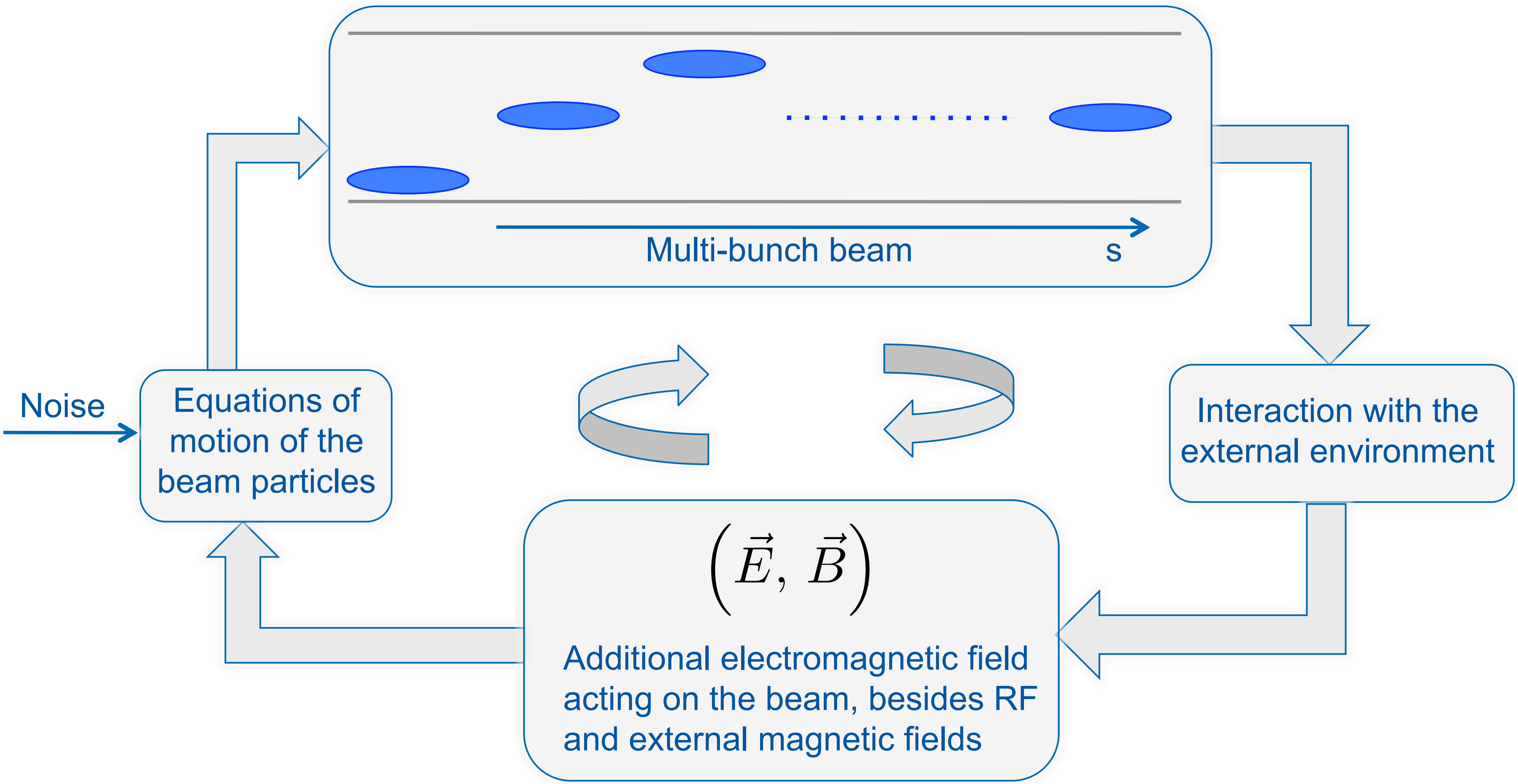


Introduction to the general problem



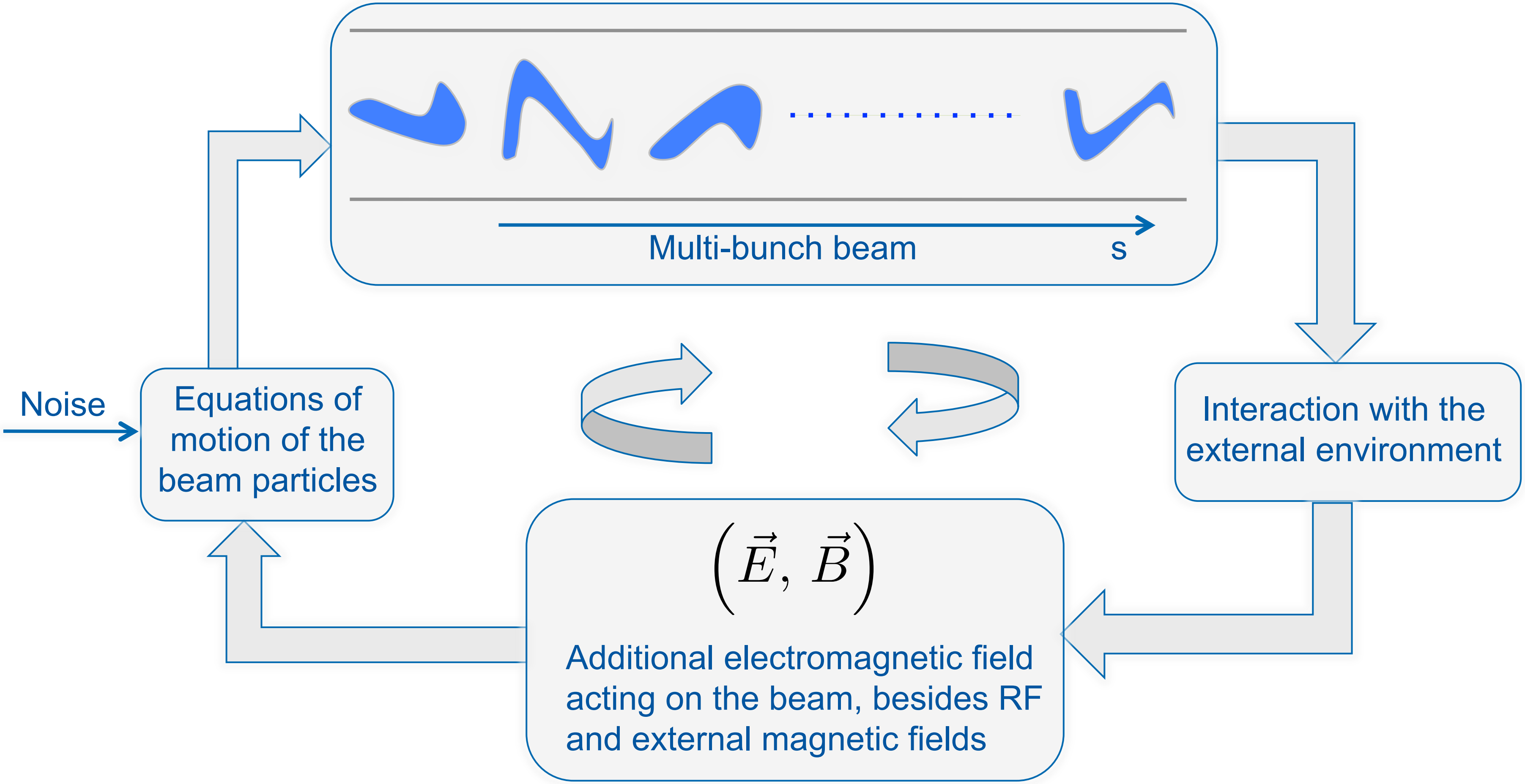
When the loop closes, either the beam will find a new stable equilibrium configuration …

Introduction to the general problem



… or it might develop an instability along the bunch train …

Introduction to the general problem



… or also an instability affecting different bunches independently of each other

Introduction to the general problem

Interaction of the beam with the external environment

Beam self-fields

- Set of Maxwell's equations with
  - The beam as the source term
  - Boundary conditions given by the accelerator component in which the beam propagates

Fields from another beam

- Set of Maxwell's equations with
  - The other beam as the source term
  - Boundary conditions given by the chamber in which the encounter between the beams takes place

$\left(\vec{E},\vec{B}\right)$

Additional electromagnetic field acting on the beam, besides RF and external magnetic fields

The electron/ion cloud

- Electron/ion production and accumulation
- Poisson's equation with
  - The electron cloud as the source term
  - Boundary conditions given by the chamber in which the electron cloud builds up

Introduction to the general problem

Interaction of the beam with the external environment

Beam self-fields

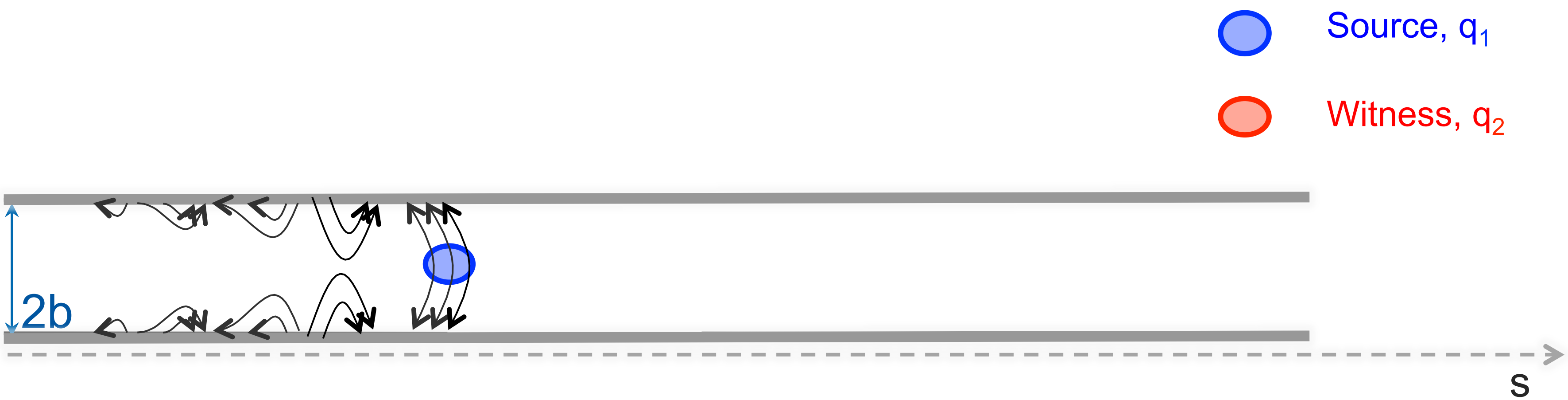
- Set of Maxwell's equations with
  - The beam as the source term
  - Boundary conditions given by the accelerator component in which the beam propagates

Need of a mathematical framework to describe the effects of the self-induced fields on the beam

Time domain: Wake fields

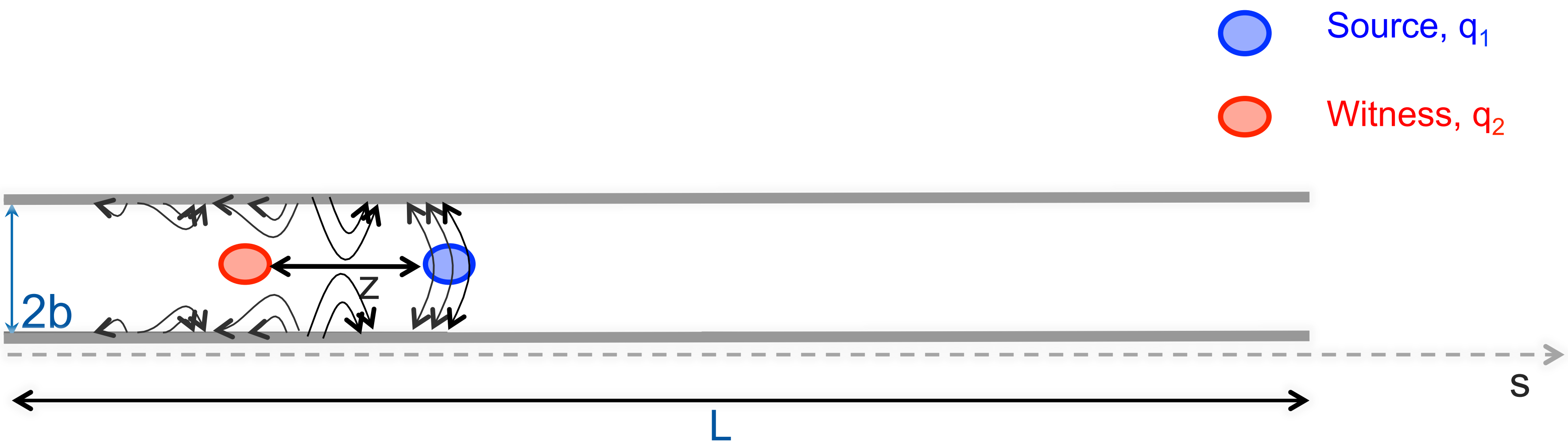
Frequency domain: Beam coupling impedances

General concept of wake field



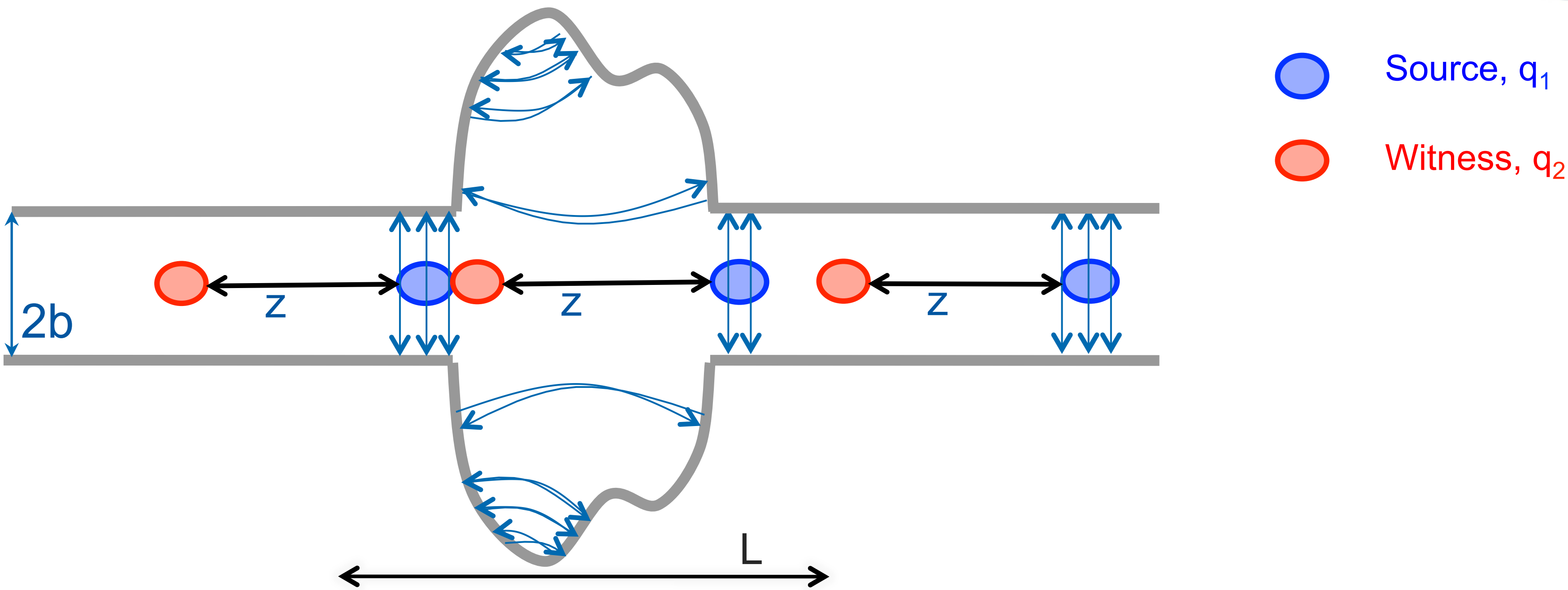
– A charge $q_1\delta(s\text{-}ct)$ traveling down a pipe with finite conductivity induces delayed currents in the wall and a trailing electromagnetic field

General concept of wake field



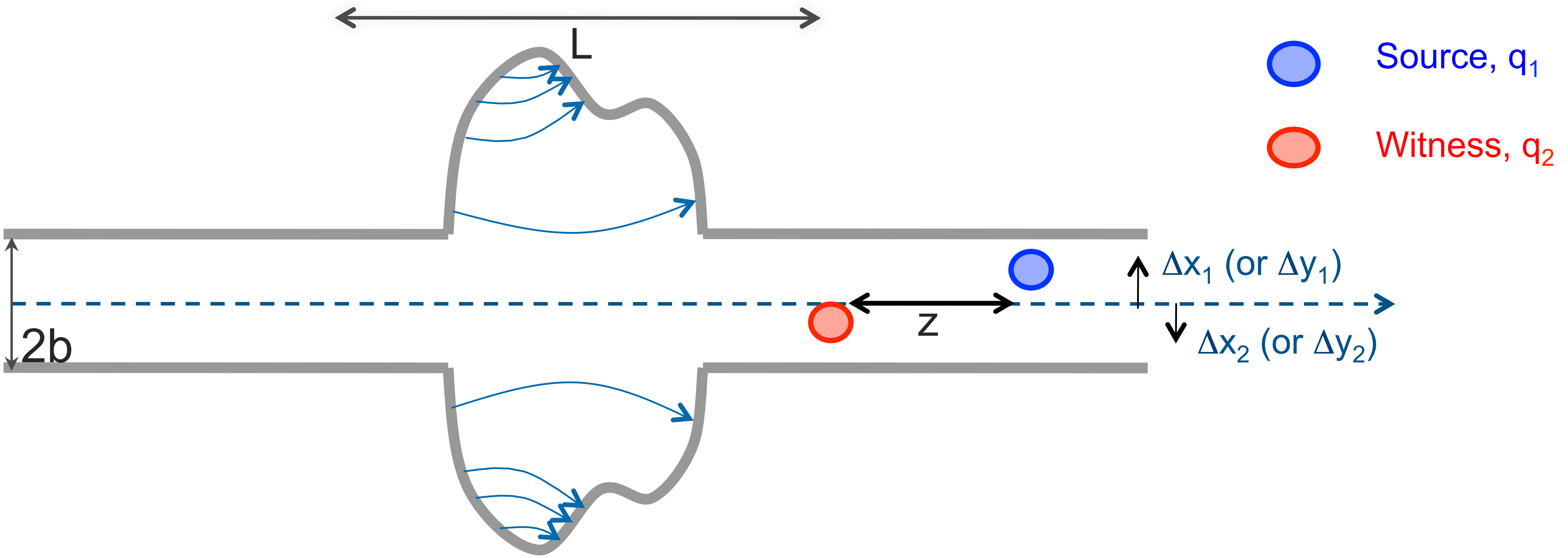
- A charge $q_1\delta(s-ct)$ traveling down a pipe with finite conductivity induces delayed currents in the wall and a trailing electromagnetic field
  - A witness charge $q_2\delta(s-ct-z)$ at a distance z from the source feels the effect of this electromagnetic field
  - Due to translational symmetry, the field acting on the witness only depends on z, and not on s and t separately
  - The force all along a certain length of the structure, L (which will define the wake function associated to the the length L of beam chamber) is constant
- In general, all electromagnetic boundary conditions other than PEC, but also geometric discontinuities, are the origin of wake fields.

General concept of wake field



- While source and witness, distant by *z<0*, move in a perfectly conducting chamber, the witness does not feel any force (γ >> 1)
- When the source encounters a discontinuity (e.g., transition, device), the new electromagnetic field configuration that satisfies the boundary conditions will trail behind (wake field) and can ring for long if modes are trapped in the structure and losses are low
  - The source loses energy upon crossing the discontinuity
  - The witness feels a net force all along an effective length of the structure, L

Longitudinal wake function



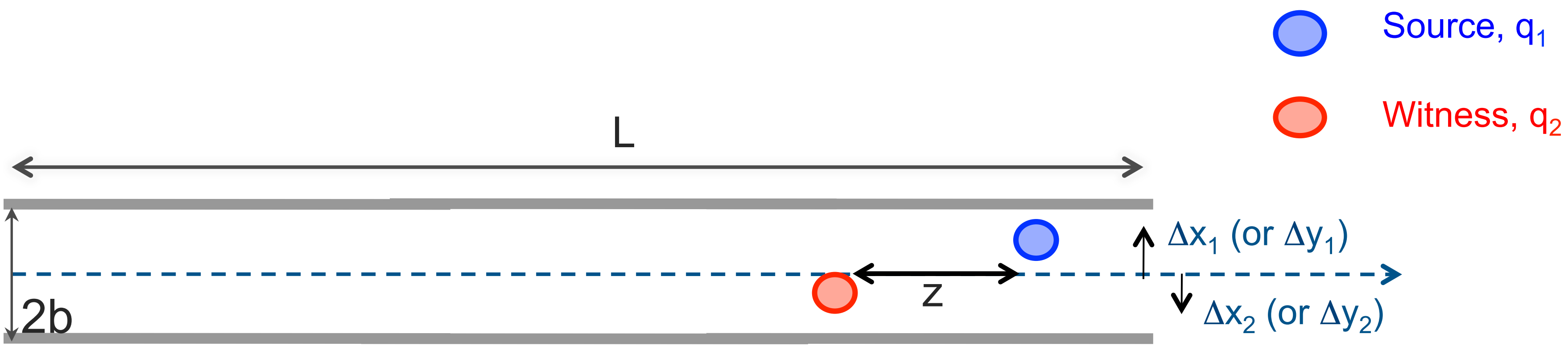
Since there is no translational symmetry, the force on the witness charge depends on both s and t, or its distance z to the source (and source and witness positions):

$$F_s(s,z) = q_2 E_s(s,z) \quad \rightarrow \quad F_s(s,z;\Delta x_1,\Delta x_2) = q_2 E_s(s,z;\Delta x_1,\Delta x_2)$$

But we can still integrate this force along the path where $E_s \neq 0$ and get rid of the s dependence

$$\int_0^L F_s(z;\Delta x_1,\Delta x_2) ds$$

Longitudinal wake function



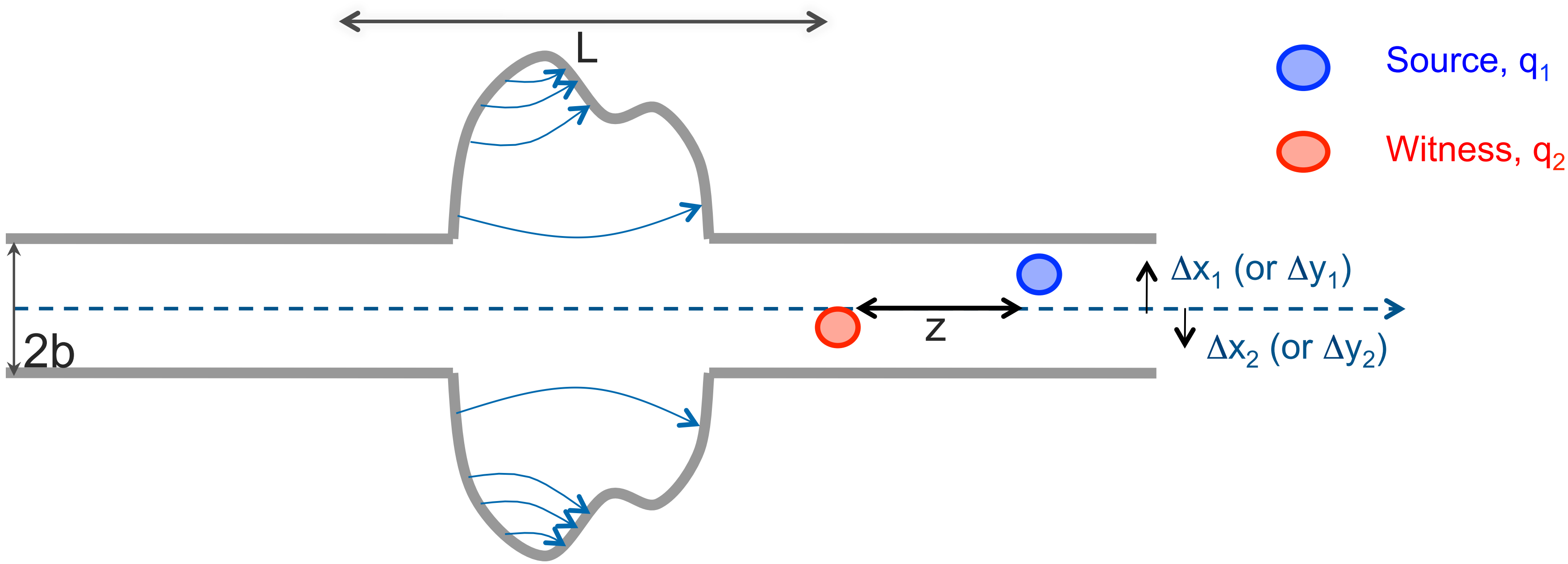
In the special case of a segment of length L of resistive beam chamber

$$F_s(z; \Delta x_1, \Delta x_2) = q_2 E_s(z; \Delta x_1, \Delta x_2)$$

And the integration simply yields

$$\int_0^L F_s(z; \Delta x_1, \Delta x_2) ds = q_2 E_s(z; \Delta x_1, \Delta x_2) L$$

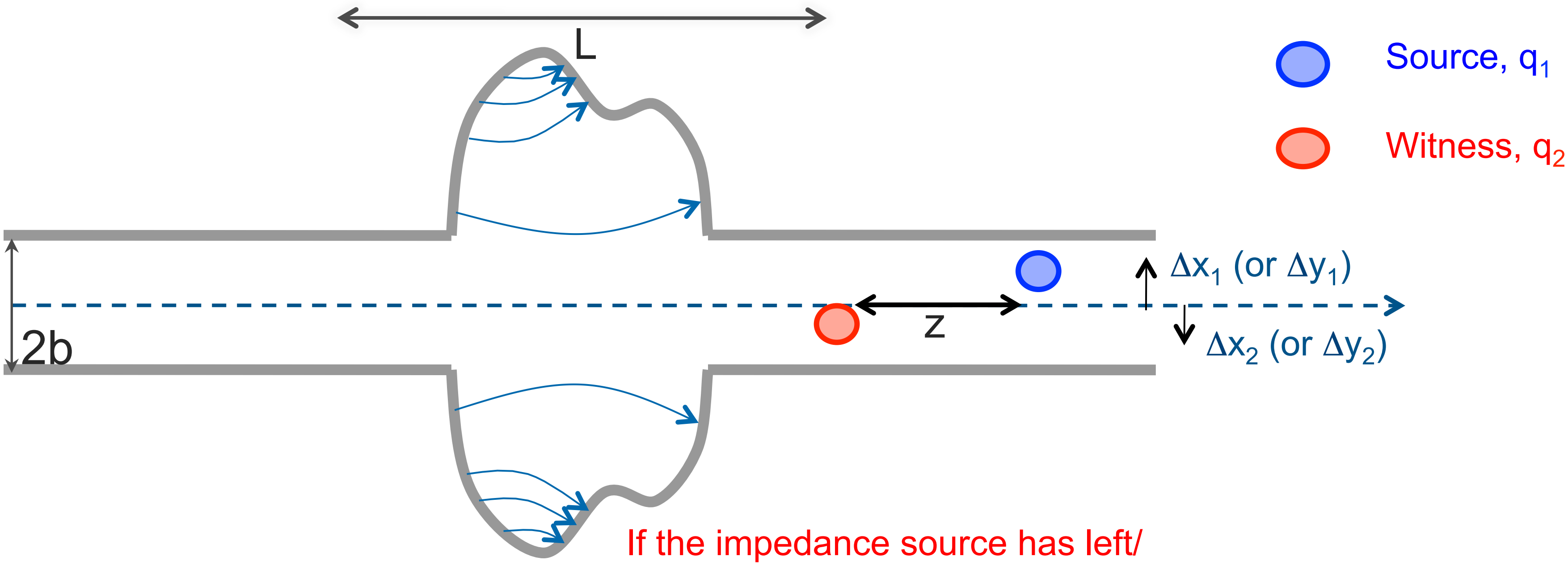
Longitudinal wake function



$$\int_0^L F_s(s,z)ds = -q_1 q_2 \left[W_{||}(z) + W_{||}^{(1d)}(z)\Delta x_1 + W_{||}^{(1q)}(z)\Delta x_2 + W_{||}^{(2d)}(z)\Delta x_1^2 + W_{||}^{(2q)}(z)\Delta x_2^2 \right.$$

$$\left. + W_{||}^{(dq)}(z)\Delta x_1 \Delta x_2 + \ldots \right]$$

Longitudinal wake function



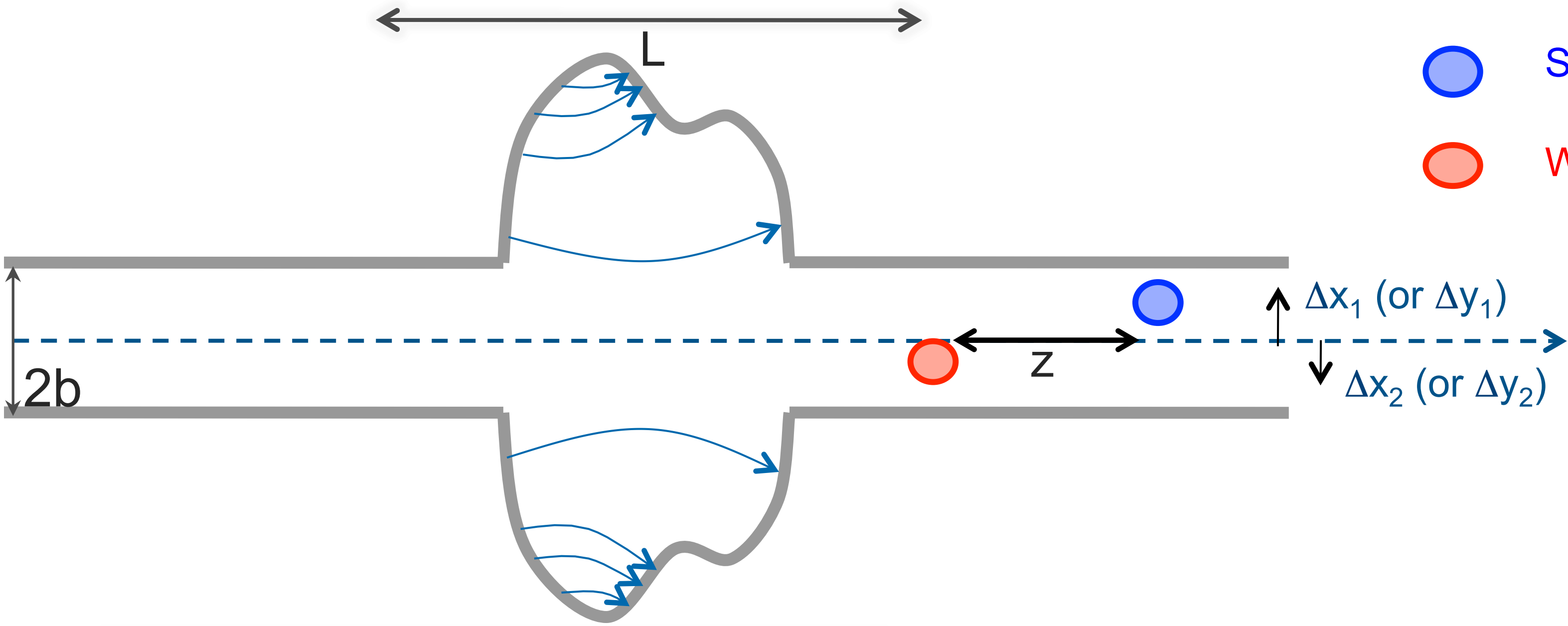
If the impedance source has left/ right top/bottom symmetry

$$\int_0^L F_s(s,z)ds = -q_1q_2\left[W_{||}(z) + W_{||}^{(1d)}(z)\Delta x_1 + W_{||}^{(1q)}(z)\Delta x_2 + W_{||}^{(2d)}(z)\Delta x_1^2 + W_{||}^{(2q)}(z)\Delta x_2^2 + W_{||}^{(dq)}(z)\Delta x_1\Delta x_2 + \ldots\right]$$

Order zero: source and test centered. Usually dominant

Second order terms. Negligible for small offsets

Longitudinal wake function



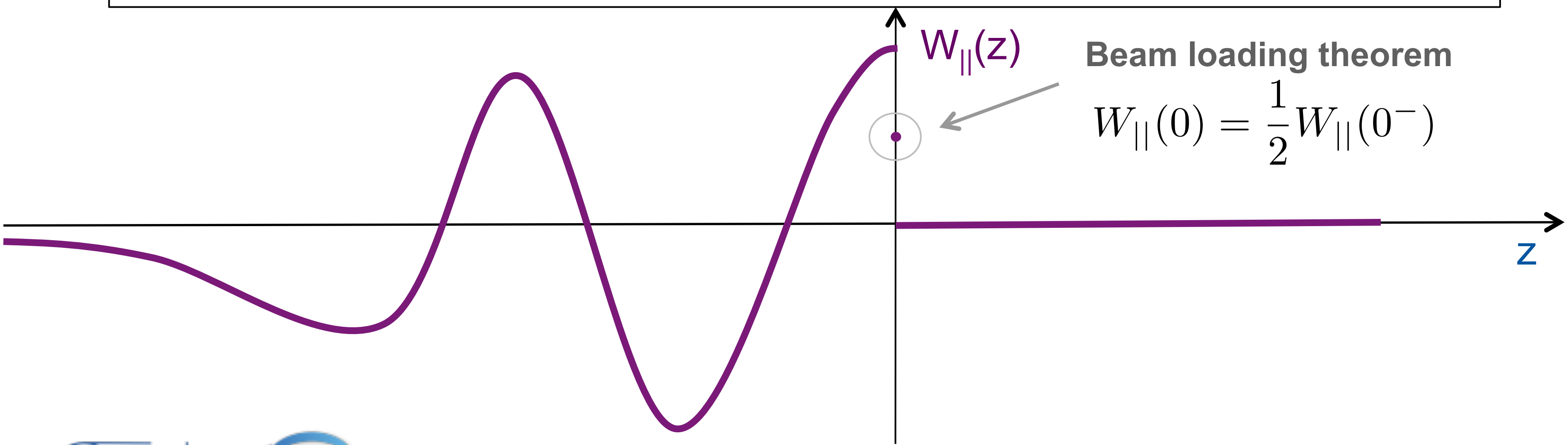
$$\int_0^L F_{\|}(s,z)ds = -q_1 q_2 W_{\|}(z)$$

$$\Delta E_2 \quad \Rightarrow \quad \frac{\Delta E_2}{E_0} = \left(\frac{\gamma^2 - 1}{\gamma^2}\right)\frac{\Delta p_2}{p_0}$$

Longitudinal wake function

$$W_{||}(z) = -\frac{\Delta E_2}{q_1 q_2} \qquad \begin{matrix} z \rightarrow 0 \\ q_2 \rightarrow q_1 \end{matrix} \qquad W_{||}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in 0, ***$W_{||}(0)$***, is related to the energy lost by the source particle in the creation of the wake
- ***$W_{||}(0)>0$*** since ***$\Delta E_1<0$***
- ***$W_{||}(0^-)>0$*** since the longitudinal electric field must be retarding immediately following the source, regardless of any boundary conditions
- ***$W_{||}(z)$*** is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



Longitudinal impedance

- The wake function of an accelerator component is basically its Green function in time domain (i.e., its response to a pulse excitation)
  - ⇒ Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!

- We can also describe it as a transfer function in frequency domain
- This is the definition of **longitudinal beam coupling impedance** of the element under study

$$\boxed{Z_{\parallel}(\omega)} = \int_{-\infty}^{\infty} \boxed{W_{\parallel}(z)} \exp\left(-\frac{i\omega z}{c}\right)\frac{dz}{c}$$

[Ω]

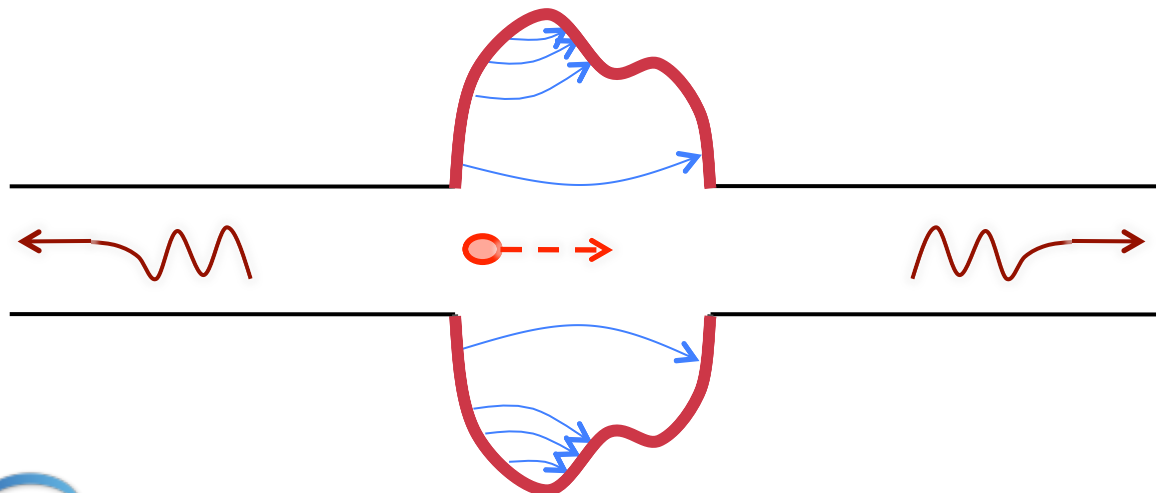
[Ω/s]

The energy balance

$$W_{||}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}[Z_{||}(\omega)]d\omega = \frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into
  - Electromagnetic energy of the modes that remain trapped in the object
    - ⇒ Partly dissipated on lossy walls or into purposely designed inserts or HOM absorbers
    - ⇒ Partly transferred to following particles (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of modes that propagate down the beam chamber (above cut-off), eventually lost on surrounding lossy materials



The energy balance

$$W_{||}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}[Z_{||}(\omega)]d\omega = \frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

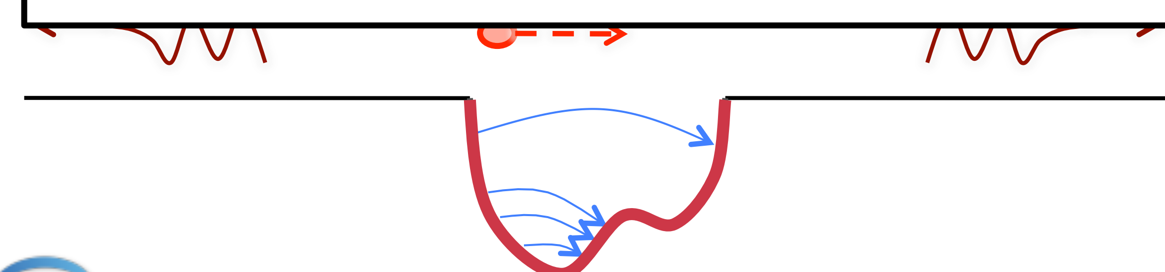
- In the global energy balance, the energy lost by the source splits into
  - Electromagnetic energy of the modes that remain trapped in the object
  - ...nserts
  - ...over
  - E... c... ...terials

The energy loss of a particle bunch

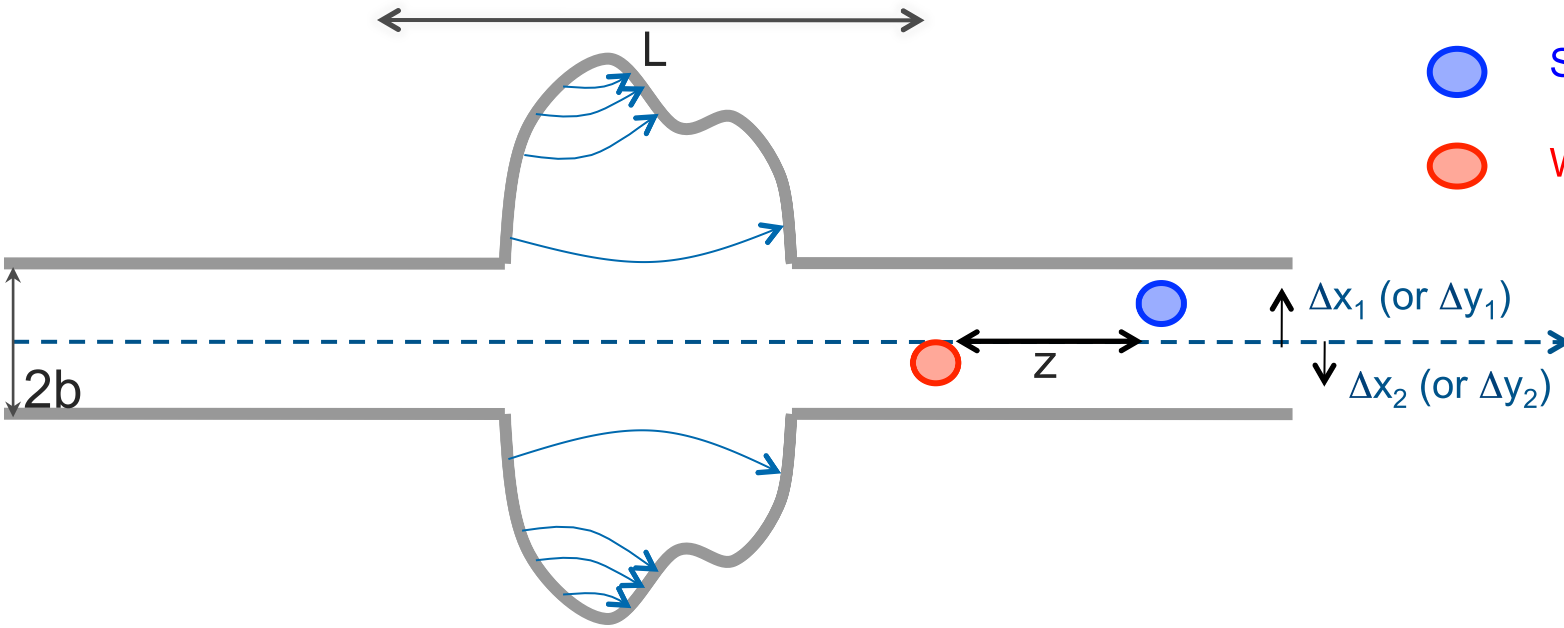
⇒ Causes **beam induced heating** of the machine elements (damage, outgassing)

⇒ Is compensated by the RF system determining a stable phase shift

⇒ Feeds into both **longitudinal and transverse instabilities** through the associated EM fields



Transverse wake function

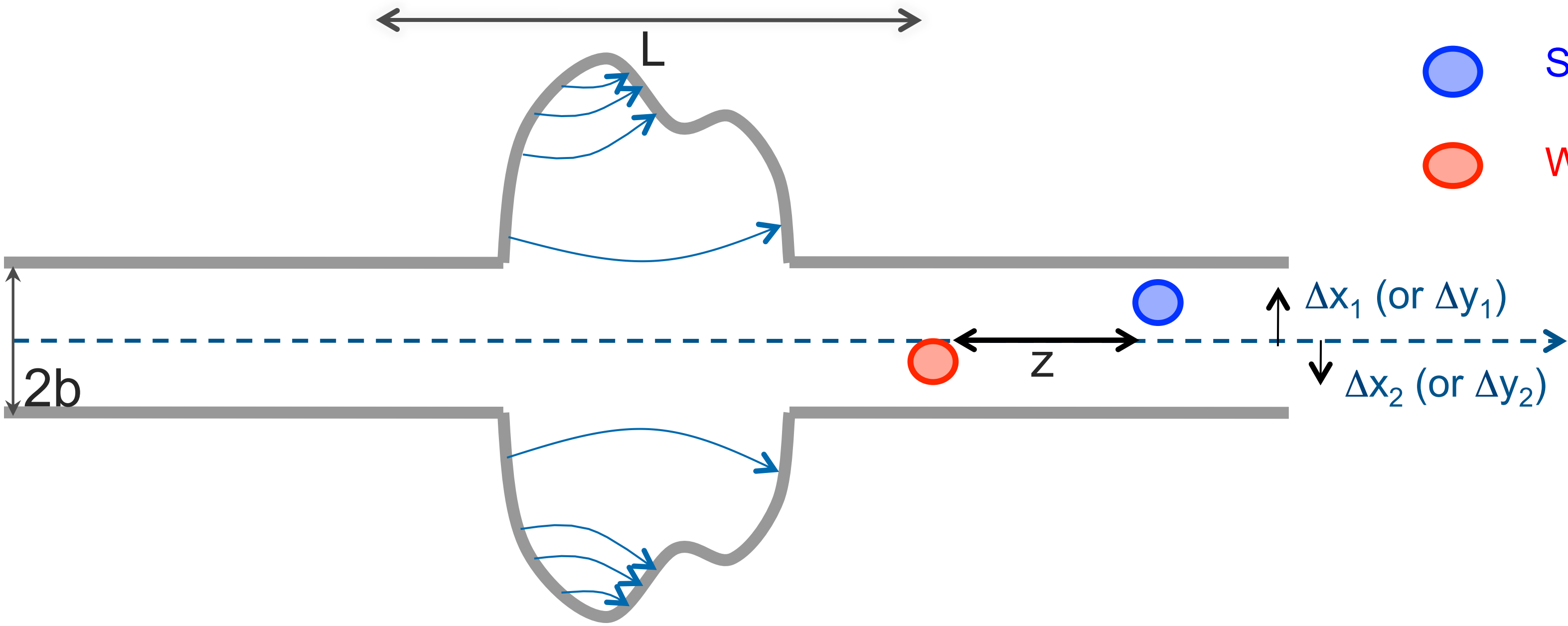


Also in the transverse plane, the force on the witness charge depends on both s and t, or its distance z to the source (and source and witness positions):

$$F_{x,y}(s,z;\Delta x_1,\Delta x_2) = q_2\left[\vec{E}(s,z;\Delta x_1,\Delta x_2) + \vec{v}\times\vec{B}(s,z;\Delta x_1,\Delta x_2)\right]_{x,y}$$

If we neglect the contributions $v_x B_s$ and $v_y B_s$ ($v_{x,y} \ll v_s$)

Transverse wake function



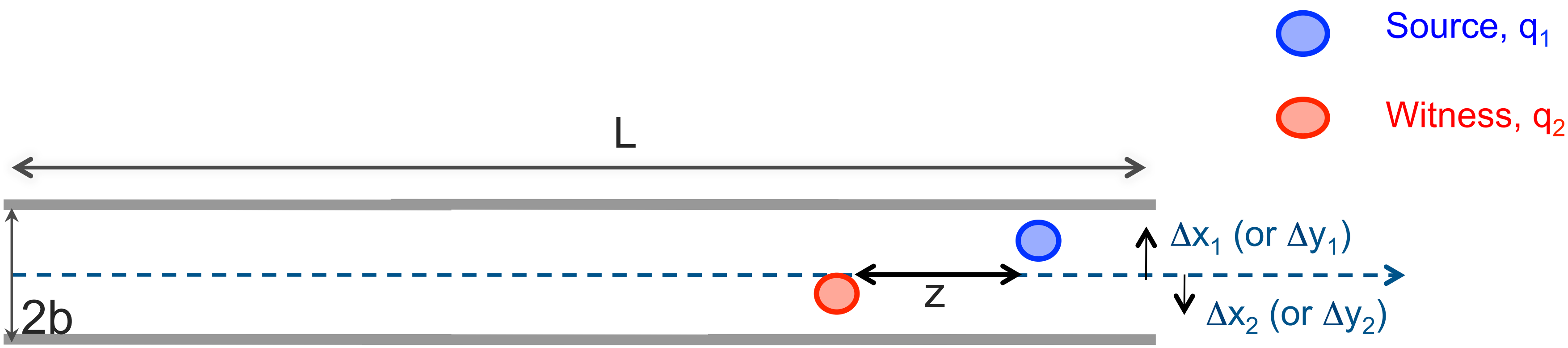
Also in the transverse plane, the force on the witness charge depends on both s and t, or its distance z to the source (and source and witness positions):

$$F_{x,y}(s,z;\Delta x_1,\Delta x_2) = q_2\left[E_{x,y}(s,z;\Delta x_1,\Delta x_2) \mp vB_{y,x}(s,z;\Delta x_1,\Delta x_2)\right]$$

We can also integrate this force along the path where $F_{x,y} \neq 0$ and get rid of the s dependence

$$\int_0^L F_{x,y}(s,z;\Delta x_1,\Delta x_2)ds$$

Transverse wake function



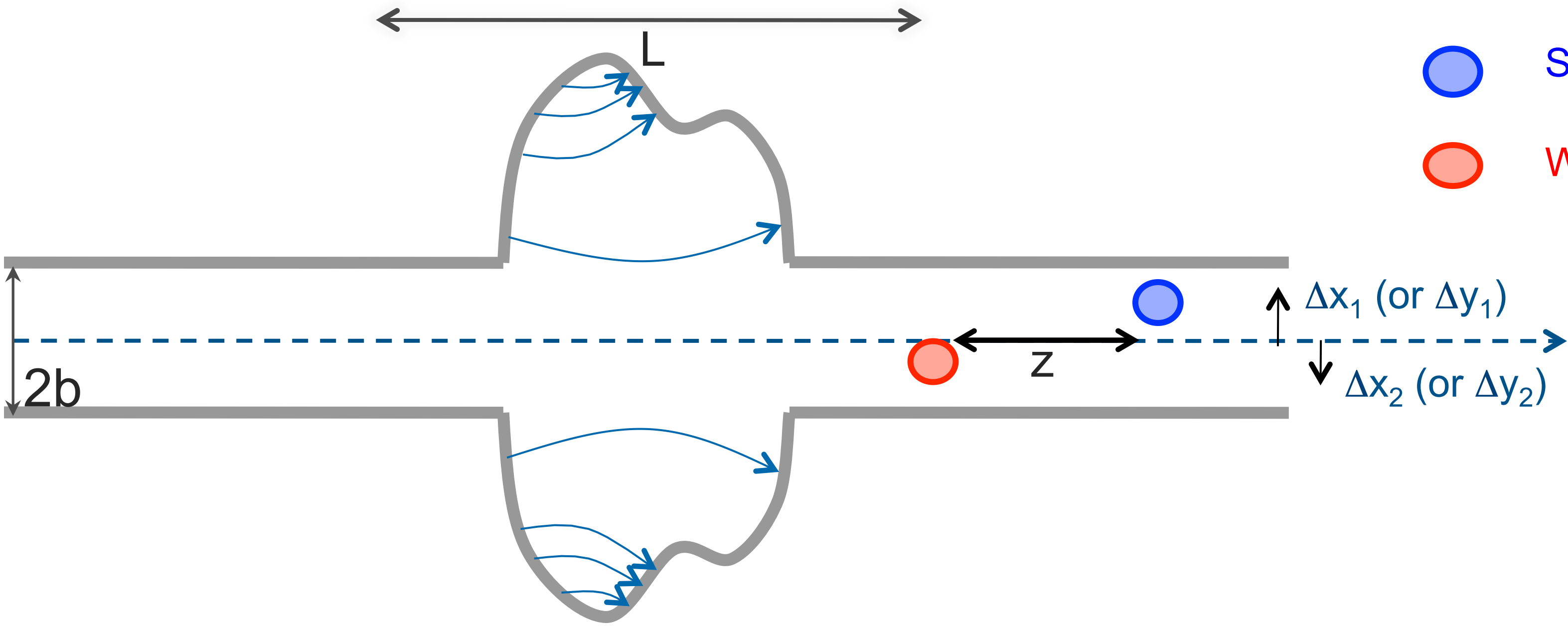
In the special case of a segment of length L of resistive beam chamber

$$F_{x,y}(z;\Delta x_1,\Delta x_2)=q_2\left[E_{x,y}(z;\Delta x_1,\Delta x_2)\mp vB_{y,x}(z;\Delta x_1,\Delta x_2)\right]$$

And the integration simply yields

$$\int_0^L F_{x,y}(z;\Delta x_1,\Delta x_2)ds=q_2\left[E_{x,y}(z;\Delta x_1,\Delta x_2)\mp vB_{y,x}(z;\Delta x_1,\Delta x_2)\right]L$$

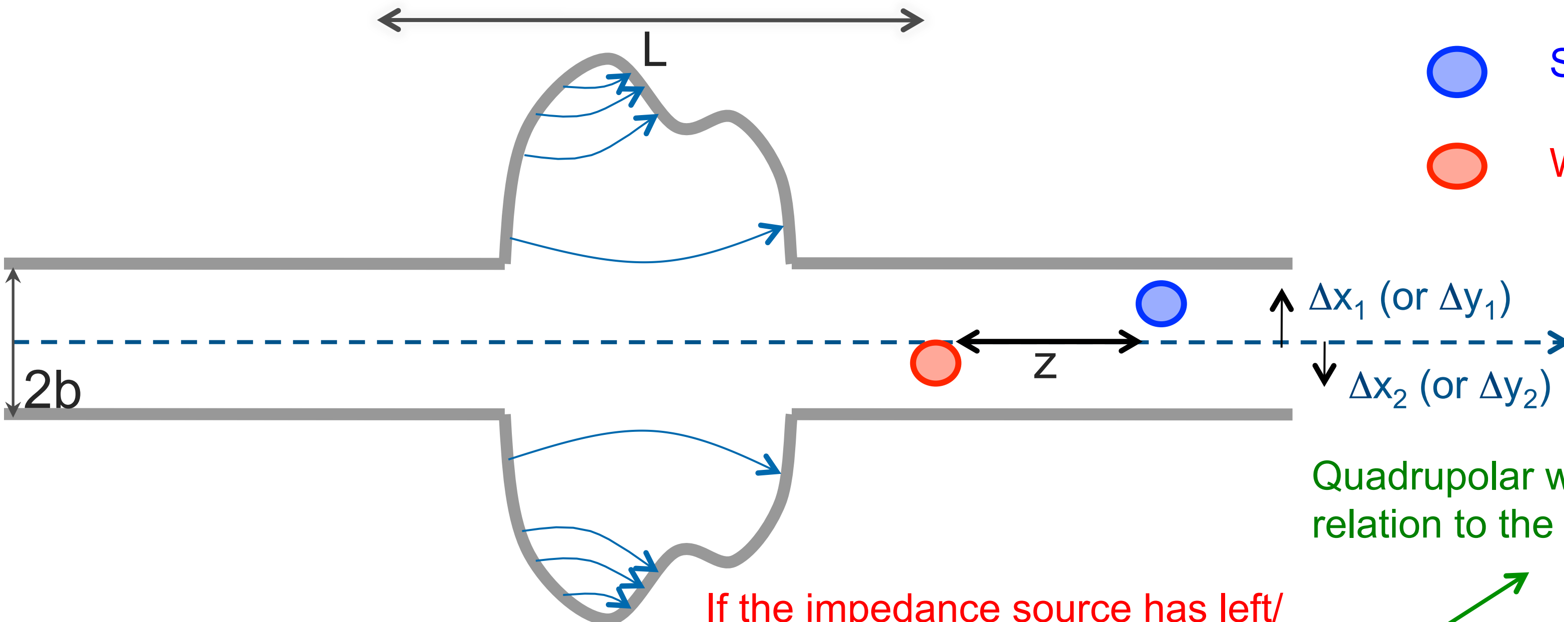
Transverse wake function



$$\int_0^L F_x(s,z)ds = -q_1 q_2 \left[W_{Cx}(z) + W_x(z)\Delta x_1 + W_{Qx}(z)\Delta x_2\right]$$

$$\int_0^L F_y(s,z)ds = -q_1 q_2 \left[W_{Cy}(z) + W_y(z)\Delta y_1 + W_{Qy}(z)\Delta y_2\right]$$

Transverse wake function



Quadrupolar wakes, express relation to the offset of the test

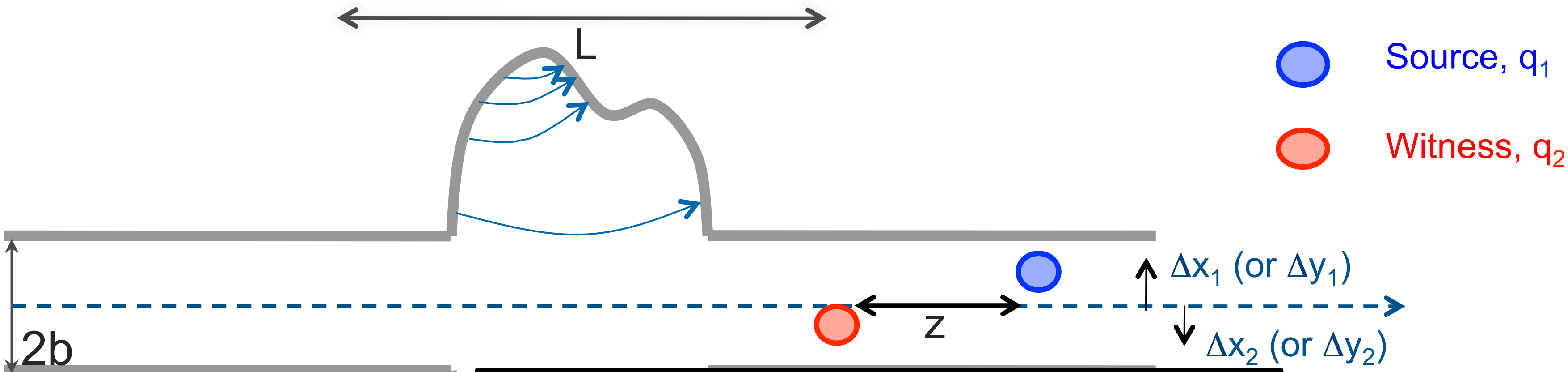
If the impedance source has left/right top/bottom symmetry

$$\int_0^L F_x(s,z)ds = -q_1 q_2 \left[W_{Cx}(z) + W_x(z)\Delta x_1 + W_{Qx}(z)\Delta x_2\right]$$

$$\int_0^L F_y(s,z)ds = -q_1 q_2 \left[W_{Cy}(z) + W_y(z)\Delta y_1 + W_{Qy}(z)\Delta y_2\right]$$

Dipolar wakes, express relation to the offset of the source

Transverse wake function



We have truncated to the first order, thus neglecting

- ⇒ First order coupling terms between x and y planes
- ⇒ All higher order terms in the wake expansion (including mixed higher order terms with products of the dipolar/ quadrupolar offsets)

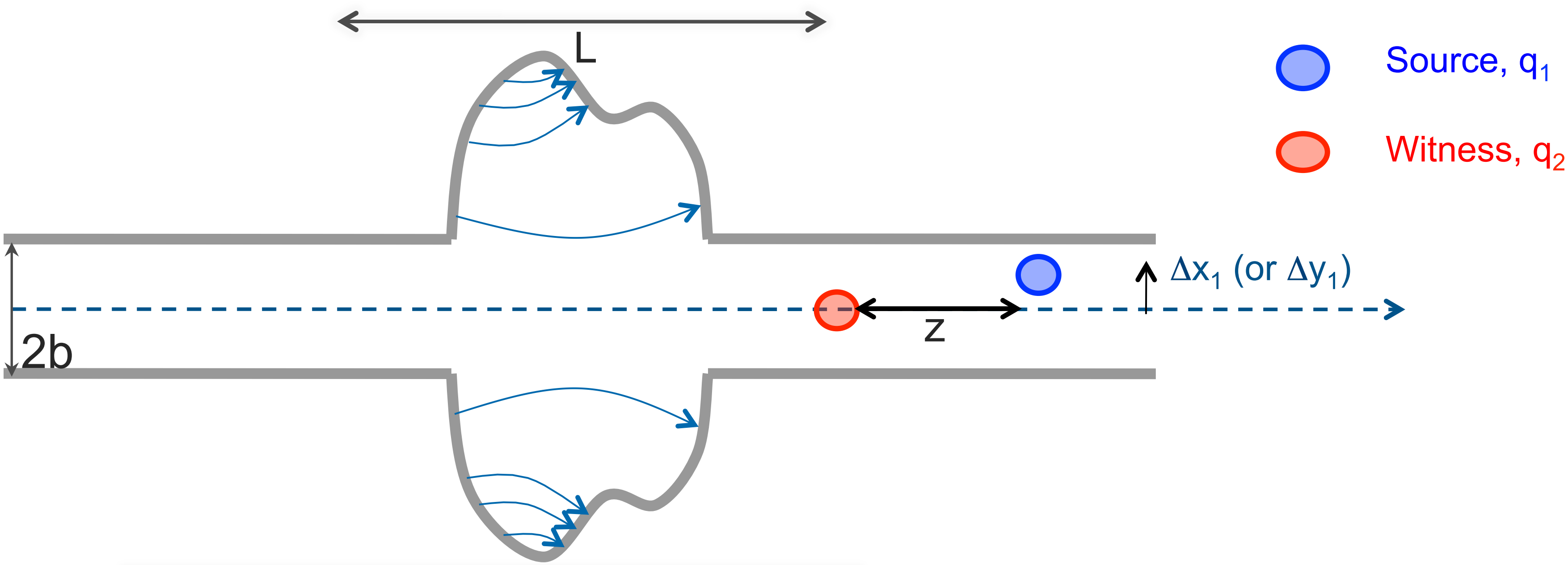
upolar wakes, express
to the offset of the test

$$\int_0^L F_x(s,z)ds = - \ldots \Delta x_2]$$

$$\int_0^L F_y(s,z)ds = - \ldots \Delta y_2]$$

Dipolar wakes, express relation to the offset of the source

Transverse wake function: dipolar



$$\int_0^L F_x(s,z)ds = -q_1 q_2 W_x(z)\Delta x_1$$

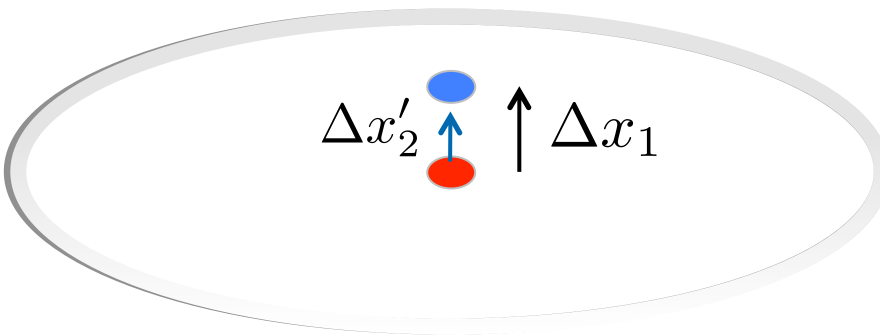
$$\int_0^L F_y(s,z)ds = -q_1 q_2 W_y(z)\Delta y_1$$

$$\Delta E_{2x,y} \quad \rightarrow \quad \frac{\Delta E_{2x,y}}{E_0} = \Delta x_2',\ \Delta y_2'$$

Transverse wake function: dipolar

$$W_x(z) = -\frac{E_0}{q_1 q_2}\frac{\Delta x'_2}{\Delta x_1} \qquad z \to 0 \qquad W_x(0) = 0$$

- The value of the transverse dipolar wake functions in 0, ***$W_{x,y}(0)$***, vanishes because source and witness particles are traveling parallel and they can only – mutually – interact through space charge, which is not included in this framework
- ***$W_{x,y}(0^{-})<0$*** since trailing particles are deflected toward the source particle (Δx_1 and $\Delta x'_2$ have the same sign)



$$\frac{\Delta x'_2}{\Delta x_1} > 0$$

Transverse wake function: dipolar

$$W_x(z) = -\frac{E_0}{q_1 q_2}\frac{\Delta x_2'}{\Delta x_1} \qquad z \to 0 \qquad W_x(0) = 0$$

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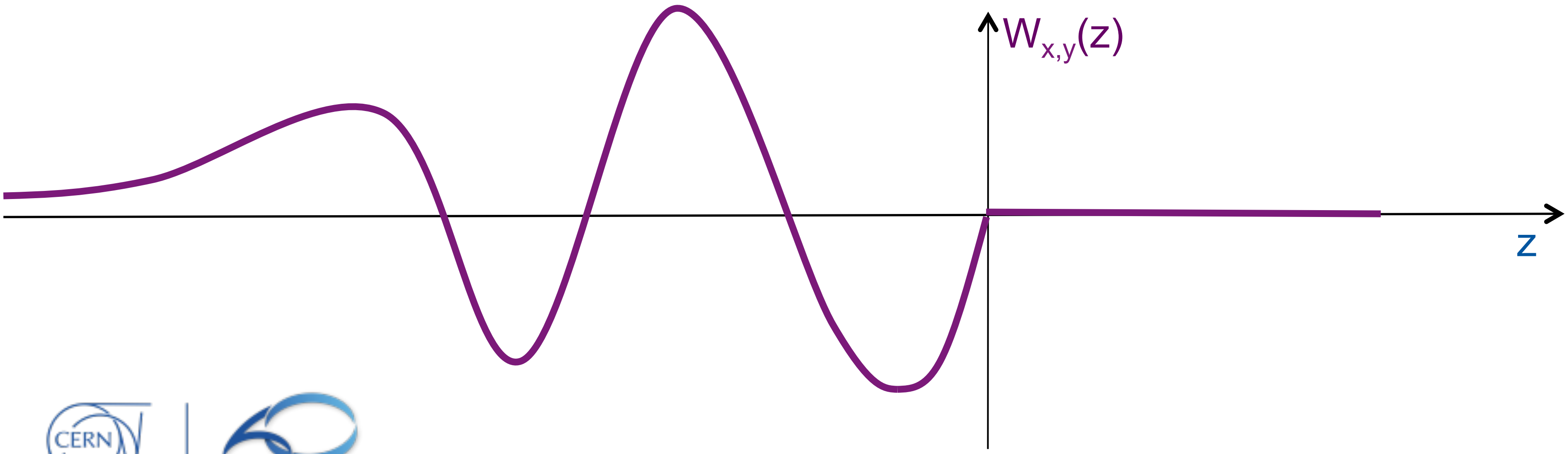
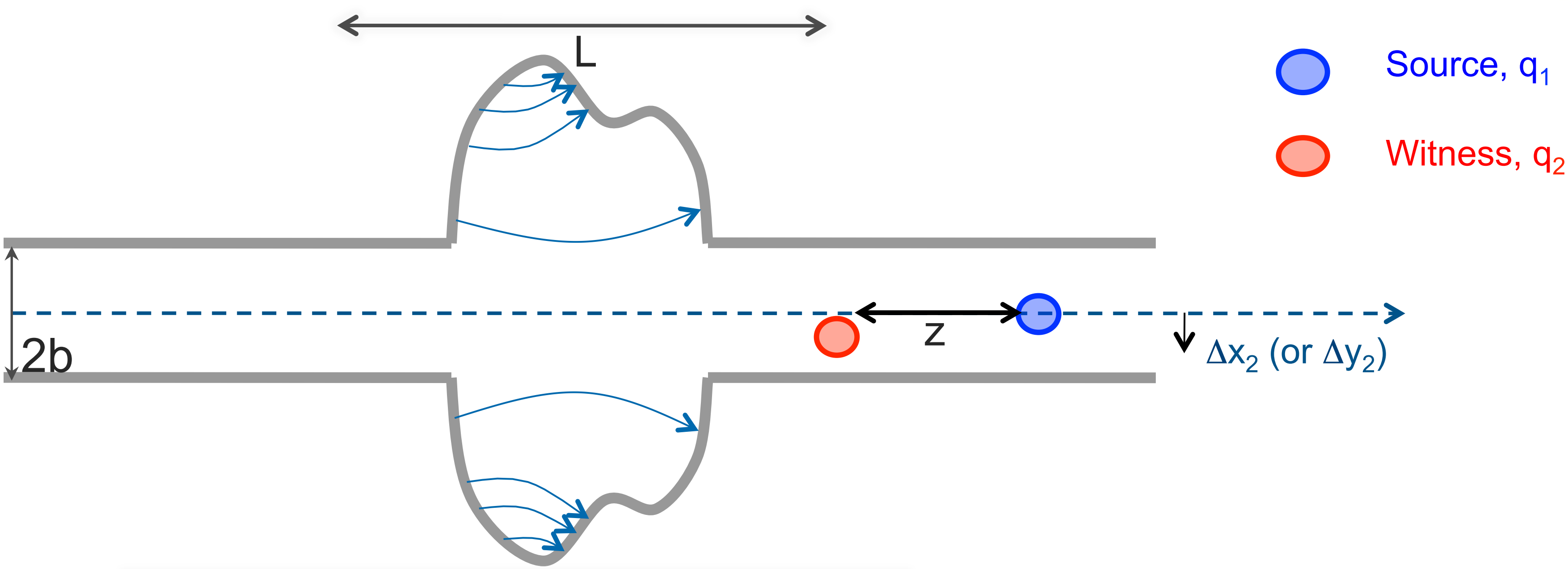
- ***$W_{x,y}(0^{-})<0$*** since trailing particles are deflected toward the source particle (Δx_1 and $\Delta x'_2$ have the same sign)
- ***$W_{x,y}(z)$*** has a discontinuous derivative in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



Transverse wake function: quadrupolar



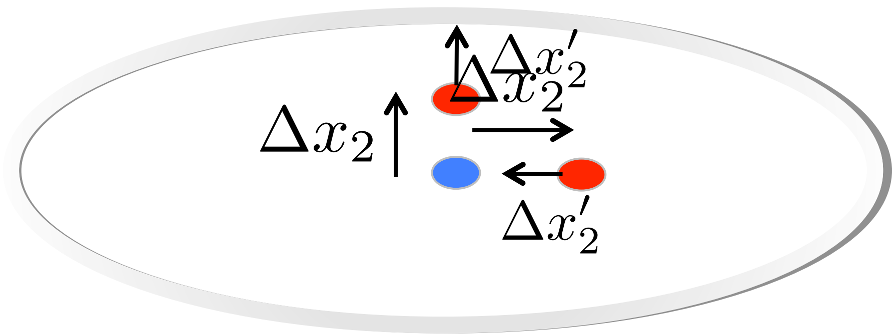
$$\int_0^L F_x(s,z)ds = -q_1 q_2 W_{Qx}(z)\Delta x_2$$
$$\int_0^L F_y(s,z)ds = -q_1 q_2 W_{Qy}(z)\Delta y_2$$

$$\Delta E_{2x,y} \quad \Rightarrow \quad \frac{\Delta E_{2x,y}}{E_0} = \Delta x_2',\ \Delta y_2'$$

Transverse wake function: quadrupolar

$$W_{Qx}(z) = -\frac{E_0}{q_1 q_2}\frac{\Delta x_2'}{\Delta x_2} \qquad \xrightarrow{z\to 0} \qquad W_{Qx}(0) = 0$$

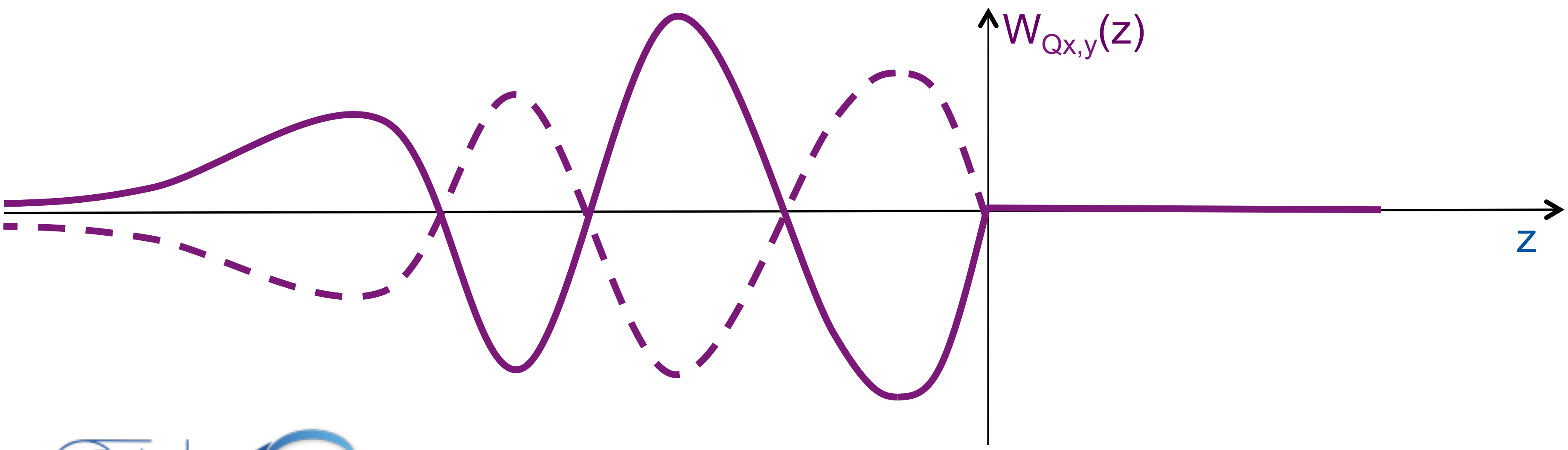
- The value of the transverse quadrupolar wake functions in 0, ***$W_{Qx,y}(0)$***, vanishes because source and witness particles are traveling parallel and they can only – mutually – interact through space charge, which is not included in this framework
- ***$W_{Qx,y}(0^{-})$*** can be of either sign since trailing particles can be either attracted or deflected even more off axis (depends on geometry and boundary conditions)



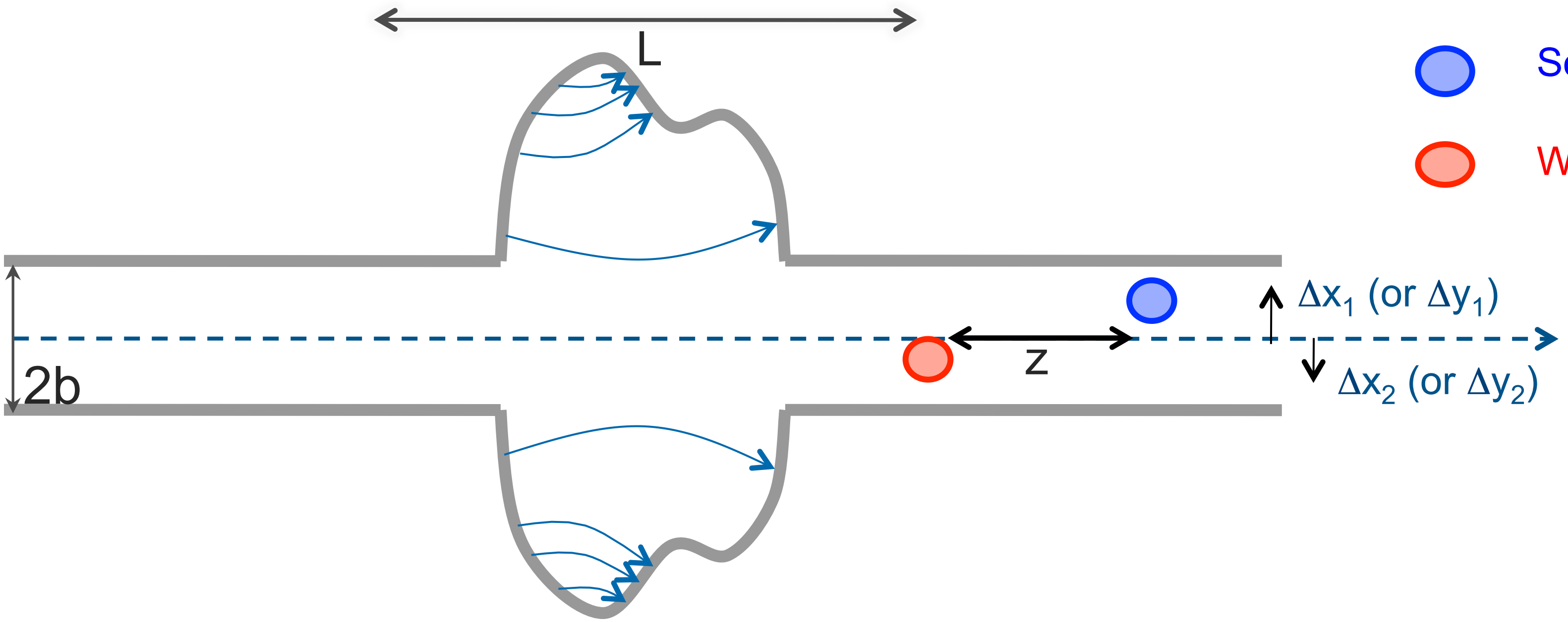
Transverse impedances

$$W_{Qx}(z) = -\frac{E_0}{q_1 q_2} \frac{\Delta x_2'}{\Delta x_2} \qquad \xrightarrow{z \to 0} \qquad W_{Qx}(0) = 0$$

- The value of the transverse quadrupolar wake functions in 0, $\boldsymbol{W_{Qx,y}(0)}$, vanishes because source and witness particles are traveling parallel and they can only – mutually – interact through space charge, which is not included in this framework
- $\boldsymbol{W_{Qx,y}(0^-)}$ can be of either sign since trailing particles can be either attracted or deflected even more off axis (depends on geometry and boundary conditions)
- $\boldsymbol{W_{x,y}(z)}$ has a discontinuous derivative in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



Transverse wake function



Dipolar

$$Z_x(\omega) = i \int_{-\infty}^{\infty} W_x(z) \exp\left(-\frac{i\omega z}{c}\right) dz$$

$$Z_y(\omega) = i \int_{-\infty}^{\infty} W_y(z) \exp\left(-\frac{i\omega z}{c}\right) dz$$

Quadrupolar

$$Z_{Qx}(\omega) = i \int_{-\infty}^{\infty} W_{Qx}(z) \exp\left(-\frac{i\omega z}{c}\right) dz$$

$$Z_{Qy}(\omega) = i \int_{-\infty}^{\infty} W_{Qy}(z) \exp\left(-\frac{i\omega z}{c}\right) dz$$

[Ω/m]

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Source terms (displaced point charge traveling along s with speed v) in Cartesian coordinates:

$$\rho(x, y, s, t) = q_1 \delta(x - x_1) \delta(y - y_1) \delta(s - vt)$$

$$\vec{j}(x, y, s, t) = \rho(x, y, s, t) \vec{v}$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Source terms (displaced point charge traveling along s with speed v) in cylindrical coordinates:

$$\rho(r,\theta,s,t) = \frac{q_1}{r_1}\delta(r-r_1)\delta_P(\theta)\delta(s-vt) =$$

$$= \frac{q_1}{r_1}\delta(r-r_1)\delta(s-vt)\sum_{m=0}^{\infty}\frac{\cos m\theta}{\pi(1+\delta_{m0})}$$

$$\vec{j}(r,\theta,s,t) = \rho(r,\theta,s,t)\vec{v}$$

$$v = \beta c \ \ \text{with} \ \ \beta \approx 1$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s} = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_s}{\partial s} = 0$$

$$\frac{\partial B_s}{\partial y} - \frac{\partial B_y}{\partial s} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial s} - \frac{\partial B_s}{\partial x} - \frac{1}{c^2}\frac{\partial E_y}{\partial t} = 0$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_s}{\partial t} = \mu_0 \rho c$$

$$\frac{\partial E_s}{\partial y} - \frac{\partial E_y}{\partial s} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial s} - \frac{\partial E_s}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_s}{\partial t} = 0$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

We want to find relations between the forces on the witness charge:

$$\vec{F}_\perp = q_2[(E_x - cB_y)\hat{x} + (E_y + cB_x)\hat{y}]$$

$$F_s = q_2 E_s$$

with

$$s - ct = z$$

$$\frac{\partial}{\partial s} = \frac{\partial}{\partial z} = -\frac{1}{c}\frac{\partial}{\partial t}$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s} = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_s}{\partial s} = 0$$

$$\frac{\partial B_s}{\partial y} - \frac{\partial B_y}{\partial s} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial s} - \frac{\partial B_s}{\partial x} - \frac{1}{c^2}\frac{\partial E_y}{\partial t} = 0$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_s}{\partial t} = \mu_0 \rho c$$

We first use this set of equations

$$\frac{\partial E_s}{\partial y} - \frac{\partial E_y}{\partial s} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial s} - \frac{\partial E_s}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_s}{\partial t} = 0$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial F_x}{\partial z} - \frac{\partial F_s}{\partial x} = 0$$

$$\frac{\partial F_s}{\partial y} - \frac{\partial F_y}{\partial z} = 0$$

$$\frac{\partial \vec{F}_\perp}{\partial z} = \nabla_\perp F_s$$

$$\frac{\partial \int_0^L \vec{F}_\perp ds}{\partial z} = \nabla_\perp \int_0^L F_s ds$$

Result known as Panofsky-Wenzel theorem

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial \int_0^L \vec{F}_\perp ds}{\partial z} = \nabla_\perp \int_0^L F_s ds$$

$$\int_0^L F_x ds = W_x(z)\Delta x_1 + W_{Qx}(z)x$$

$$W_x'(z) = W_{||}^{(dq)}(z) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{\omega}{c} Z_x(\omega) = Z_{||}^{(dq)}(\omega)$$

$$W_{Qx}'(z) = 2W_{||}^{(2q)}(z) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{\omega}{c} Z_{Qx}(\omega) = 2Z_{||}^{(2q)}(\omega)$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial \int_0^L \vec{F}_\perp ds}{\partial z} = \nabla_\perp \int_0^L F_s ds$$

$$\int_0^L F_x ds = W_x(z)\Delta x_1 + W_{Qx}(z)x$$

The longitudinal and transverse wake functions are not independent, although in general no relation can be established between $W_{||}(z)$ and $W_{x,y}(z)$, which are the main wakes in the longitudinal and transverse planes, respectively.

$W'_x(z) = W$ … $^{(dq)}_{||}(\omega)$

$$W'_{Qx}(z) = 2W_{||}^{(2q)}(z) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \frac{\omega}{c} Z_{Qx}(\omega) = 2Z_{||}^{(2q)}(\omega)$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_s}{\partial s} = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_s}{\partial s} = 0$$

$$\frac{\partial B_s}{\partial y} - \frac{\partial B_y}{\partial s} - \frac{1}{c^2}\frac{\partial E_x}{\partial t} = 0$$

$$\frac{\partial B_x}{\partial s} - \frac{\partial B_s}{\partial x} - \frac{1}{c^2}\frac{\partial E_y}{\partial t} = 0$$

$$\frac{\partial E_s}{\partial y} - \frac{\partial E_y}{\partial s} + \frac{\partial B_x}{\partial t} = 0$$

$$\frac{\partial E_x}{\partial s} - \frac{\partial E_s}{\partial x} + \frac{\partial B_y}{\partial t} = 0$$

We can now use also these two sets of equations to find additional properties of the wakes

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \frac{1}{c^2}\frac{\partial E_s}{\partial t} = \mu_0 \rho c$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_s}{\partial t} = 0$$

Relation between transverse and longitudinal wakes (Panofsky-Wenzel theorem)

$$\frac{\partial F_x}{\partial x} = -\frac{\partial F_y}{\partial y}$$

$$\frac{\partial \int_0^L F_x ds}{\partial x} = -\frac{\partial \int_0^L F_y ds}{\partial y}$$

$$W_{Qx}(z) = -W_{Qy}(z)$$

This is an interesting result!
The quadrupolar wakes in x and y must be equal with opposite signs

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\frac{\partial \int_0^L F_x ds}{\partial y} = \frac{\partial \int_0^L F_y ds}{\partial x}$$

This relation means that the cross-wakes between x and y must be equal. We have so far ignored these terms in our derivations.

How are wakes and impedances computed?

→ **Analytical or semi-analytical approach**, when geometry is simple (or simplified)

- Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
- Find closed expressions or execute the last steps numerically to derive wakes and impedances

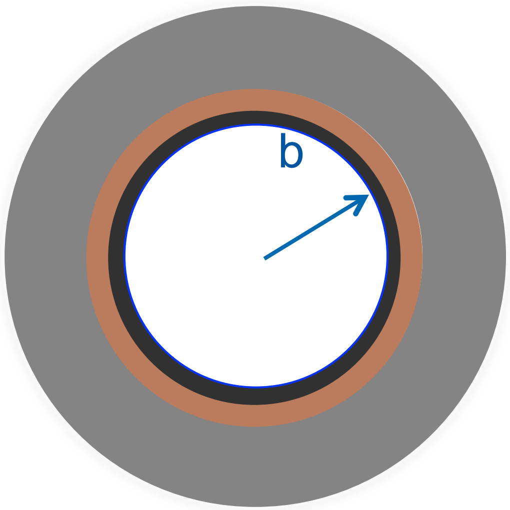
→ **Numerical approach**

- Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
- Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the ICFA mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators", Erice, Sicily, 23-28 April, 2014

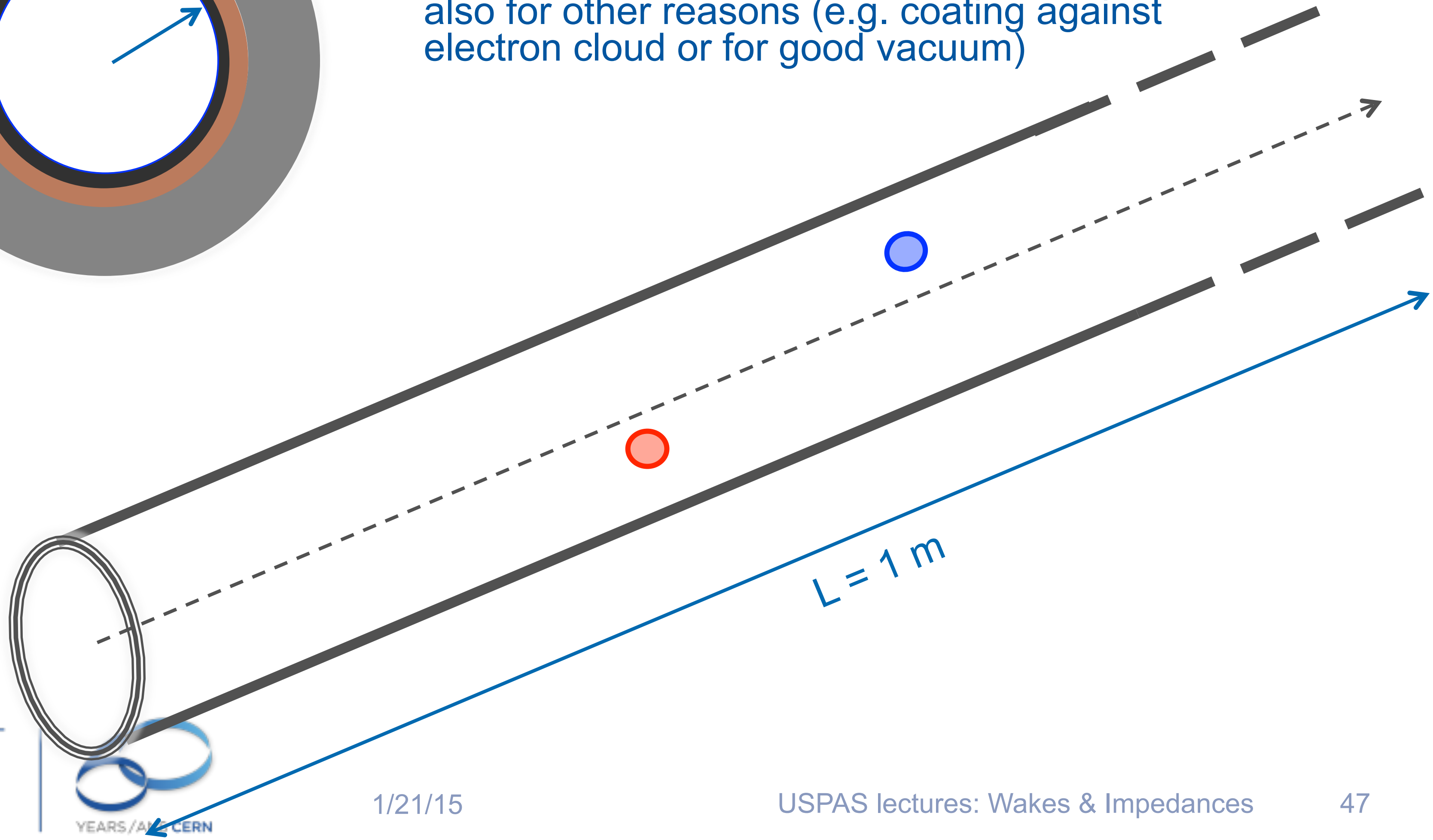
→ **Bench measurements** based on transmission/reflection measurements with stretched wires

- Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations

Examples of wakes/impedances
Resistive wall of beam chamber



- The case of a conductive pipe with an arbitrary number of layers with specified EM properties can be solved semi-analytically
- Layers sometimes required for impedance, but also for other reasons (e.g. coating against electron cloud or for good vacuum)



Examples of wakes/impedances
Resistive wall of beam chamber

$$\nabla \cdot \vec{E} = \frac{\tilde{\rho}}{\epsilon_0 \epsilon_1(\omega)} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \mu_1(\omega) \vec{J} + i\omega \frac{\mu_1(\omega)\epsilon_1(\omega)}{c^2} \vec{E} \qquad \nabla \times \vec{E} = -i\omega \vec{B}$$

Source terms (displaced point charge traveling along s with speed v) in cylindrical coordinates and frequency domain:

$$\tilde{\rho}(r,\theta,s,\omega) = \frac{q_1}{r_1 v} \delta(r - r_1) \delta_P(\theta) \exp\left(-\frac{i\omega s}{v}\right) =$$

$$= \frac{q_1}{r_1 v} \int_{-\infty}^{\infty} dk' \exp\left(-ik's\right) \delta\left(k' - \frac{\omega s}{v}\right) \sum_{m=0}^{\infty} \frac{\cos m\theta}{\pi(1+\delta_{m0})} \delta(r - r_1)$$

$$\vec{J}(r,\theta,s,\omega) = \tilde{\rho}(r,\theta,s,\omega)\vec{v}$$

Examples of wakes/impedances
Resistive wall of beam chamber

$$\nabla \cdot \vec{E} = \frac{\tilde{\rho}}{\epsilon_0 \epsilon_1(\omega)} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \mu_1(\omega) \vec{J} + i\omega \frac{\mu_1(\omega)\epsilon_1(\omega)}{c^2} \vec{E} \qquad \nabla \times \vec{E} = -i\omega \vec{B}$$

Source terms (… s with
speed v) in cylindrical coordinates and frequency domain:

Expansion in longitudinal modes

$$\tilde{\rho}(r,\theta,s,\omega) = \frac{q_1}{r_1 v}\delta(r-r_1)\delta_P(\theta)\exp\left(-\frac{i\omega s}{v}\right) =$$

$$= \frac{q_1}{r_1 v}\int_{-\infty}^{\infty} dk' \exp\left(-ik's\right)\delta\left(k' - \frac{\omega s}{v}\right) \sum_{m=0}^{\infty} \frac{\cos m\theta}{\pi(1+\delta_{m0})}\delta(r-r_1)$$

$$\vec{J}(r,\theta,s,\omega) = \tilde{\rho}(r,\theta,s,\omega)\vec{v}$$

Expansion in azimuthal modes

Examples of wakes/impedances
Resistive wall of beam chamber

Maxwell's equations combine into the wave equations:

$$\nabla^2 \vec{E} + \omega^2 \frac{\epsilon_1(\omega)\mu_1(\omega)}{c^2}\vec{E} = \frac{1}{\epsilon_0\epsilon_1(\omega)}\nabla\tilde{\rho} + i\omega\mu_o\mu_1(\omega)\tilde{\rho}\vec{v}$$

We can seek solutions as expansions of longitudinal and azimuthal modes (for both E and B)

$$\vec{E}(r,\theta,s,\omega) =$$

$$\int_{-\infty}^{\infty} dk' \exp(-ik's)\left(\sum_{m=0}^{\infty}\frac{\vec{E}^{(m,c)}(r,k',\omega)}{1+\delta_{m0}}\cos m\theta + \sum_{m=1}^{\infty}\vec{E}^{(m,s)}(r,k',\omega)\sin m\theta\right)$$

Examples of wakes/impedances
Resistive wall of beam chamber

Since we are interested in the force on the test charge, we can apply Panofsky-Wenzel and find out that we only need E_s:

$$F_s = q_2 E_s$$

$$\frac{\partial F_r}{\partial z} = \frac{\partial F_s}{\partial r} \quad \Rightarrow \quad F_r = \frac{iq_2}{k'}\frac{\partial E_s}{\partial r} \quad \Rightarrow F_r = \frac{iq_2 v}{\omega}\frac{\partial E_s}{\partial r}$$

$$\frac{\partial F_\theta}{\partial z} = \frac{1}{r}\frac{\partial F_s}{\partial \theta} \quad \Rightarrow \quad F_\theta = \frac{iq_2}{k' r}\frac{\partial E_s}{\partial \theta} \quad \Rightarrow F_\theta = \frac{iq_2 v}{\omega r}\frac{\partial E_s}{\partial \theta}$$

Examples of wakes/impedances
Resistive wall of beam chamber

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial s^2}+\frac{\omega^2}{c^2}\epsilon_1(\omega)\mu_1(\omega)\right]E_s=$$

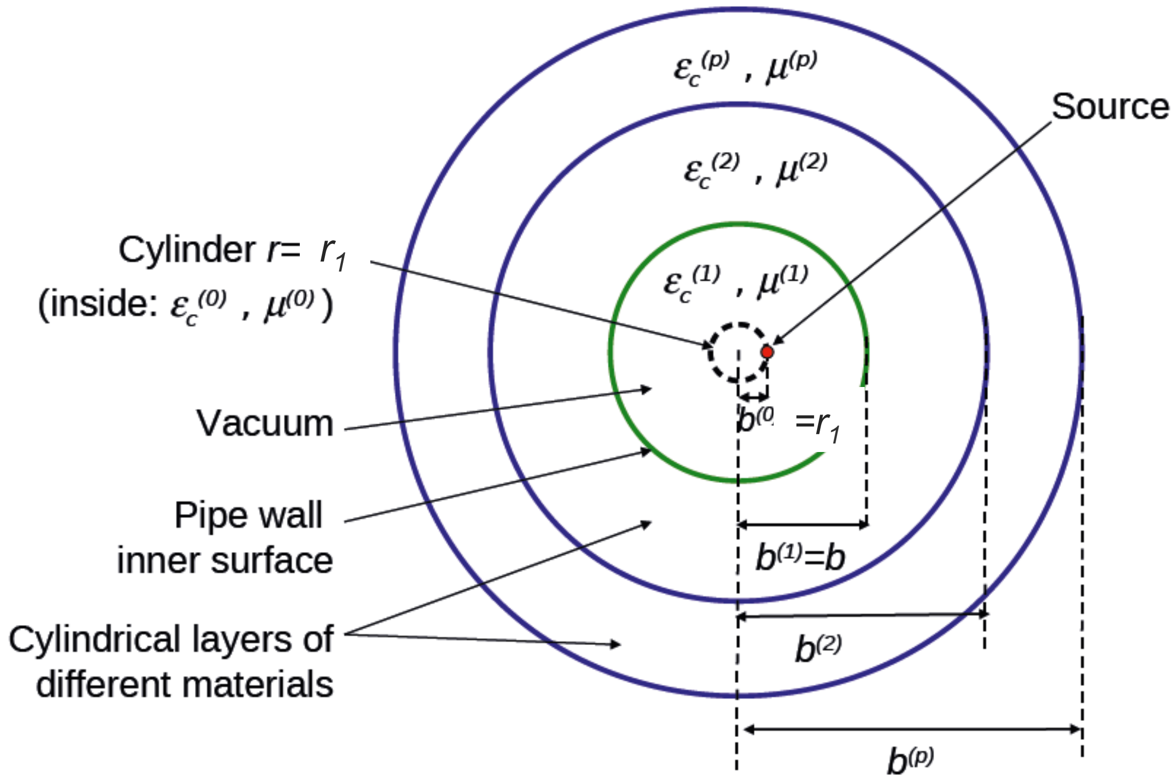
$$=\frac{1}{\epsilon_0\epsilon_1(\omega)}\frac{\partial\tilde{\rho}}{\partial s}+i\omega\mu_0\mu_1(\omega)\tilde{\rho}v$$

$$\begin{cases}\dfrac{d^2E_s^{(m,c)}}{dr^2}+\dfrac{1}{r}\dfrac{dE_s^{(m,c)}}{dr}-\left(\dfrac{m^2}{r^2}+k'^2-\dfrac{\omega^2}{c^2}\epsilon_1(\omega)\mu_1(\omega)\right)E_s^{(m,c)}= \\ \qquad =\dfrac{jq_1\delta(r-r_1)\delta(k'-\frac{\omega}{v})}{\pi r_1(1+\delta_{m0})}\left[-\dfrac{k'}{\epsilon_0\epsilon_1(\omega)}+\omega\mu_0\mu_1(\omega)\right] \\ \dfrac{d^2E_s^{(m,s)}}{dr^2}+\dfrac{1}{r}\dfrac{dE_s^{(m,s)}}{dr}-\left(\dfrac{m^2}{r^2}+k'^2-\dfrac{\omega^2}{c^2}\epsilon_1(\omega)\mu_1(\omega)\right)E_s^{(m,s)}=0\end{cases}$$

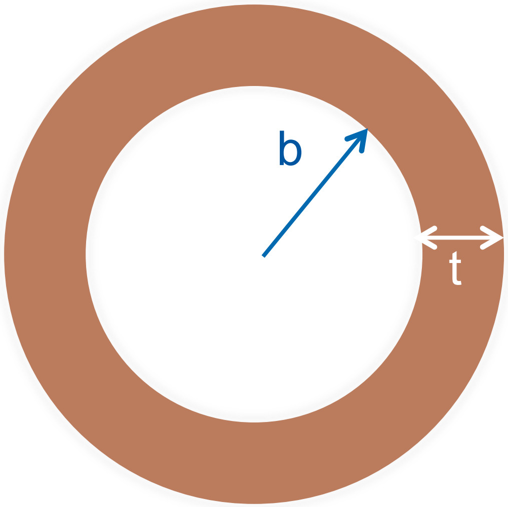
Examples of wakes/impedances
Resistive wall of beam chamber

The equations for the coefficients of the azimuthal modes of E_s must be solved in all the media and the conservation of the tangential components of the fields is applied at the boundaries between different layers

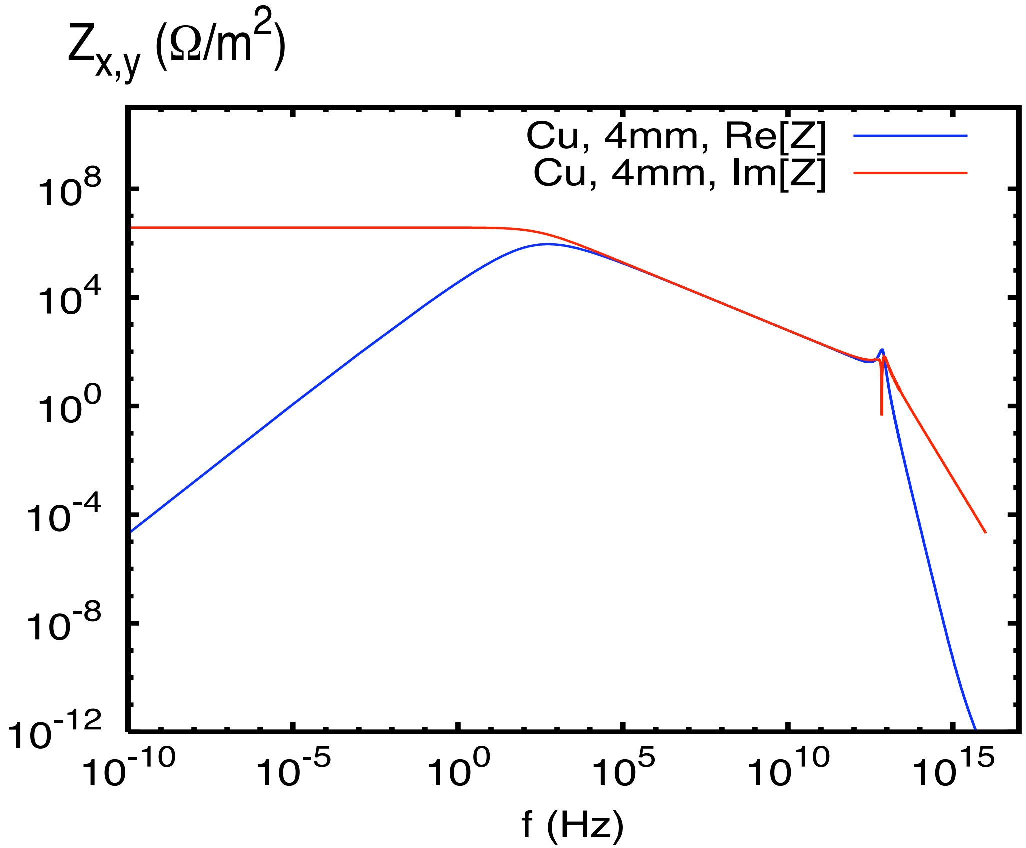
➔ E.g. ***ImpedanceWake2D*** code calculates impedances and then wakes. It can also deal with flat structures



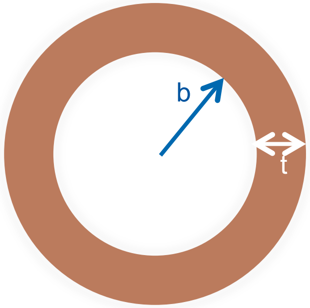
Examples of wakes/impedances
Resistive wall of beam chamber



– An interesting example: a 1 m long Cu pipe with radius *b*=2 cm and thickness *t* = 4mm in vacuum

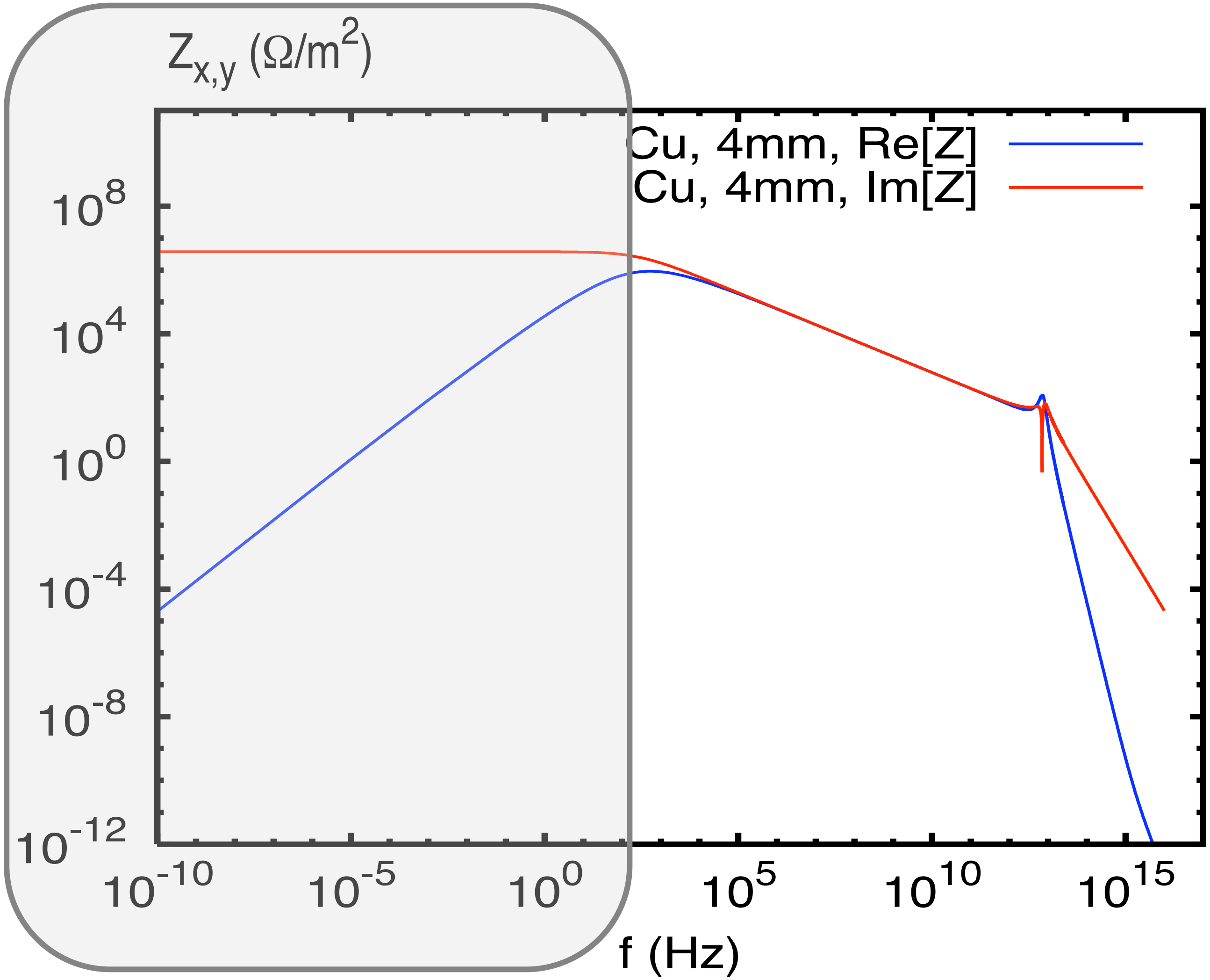


Examples of wakes/impedances
Resistive wall of beam chamber

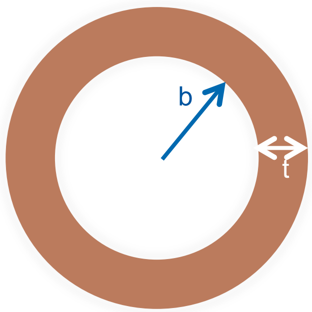


3 frequency regions of interest:

1. Below 0.1 kHz, impedance is basically purely imaginary, EM field is shielded by image charges → Indirect space charge

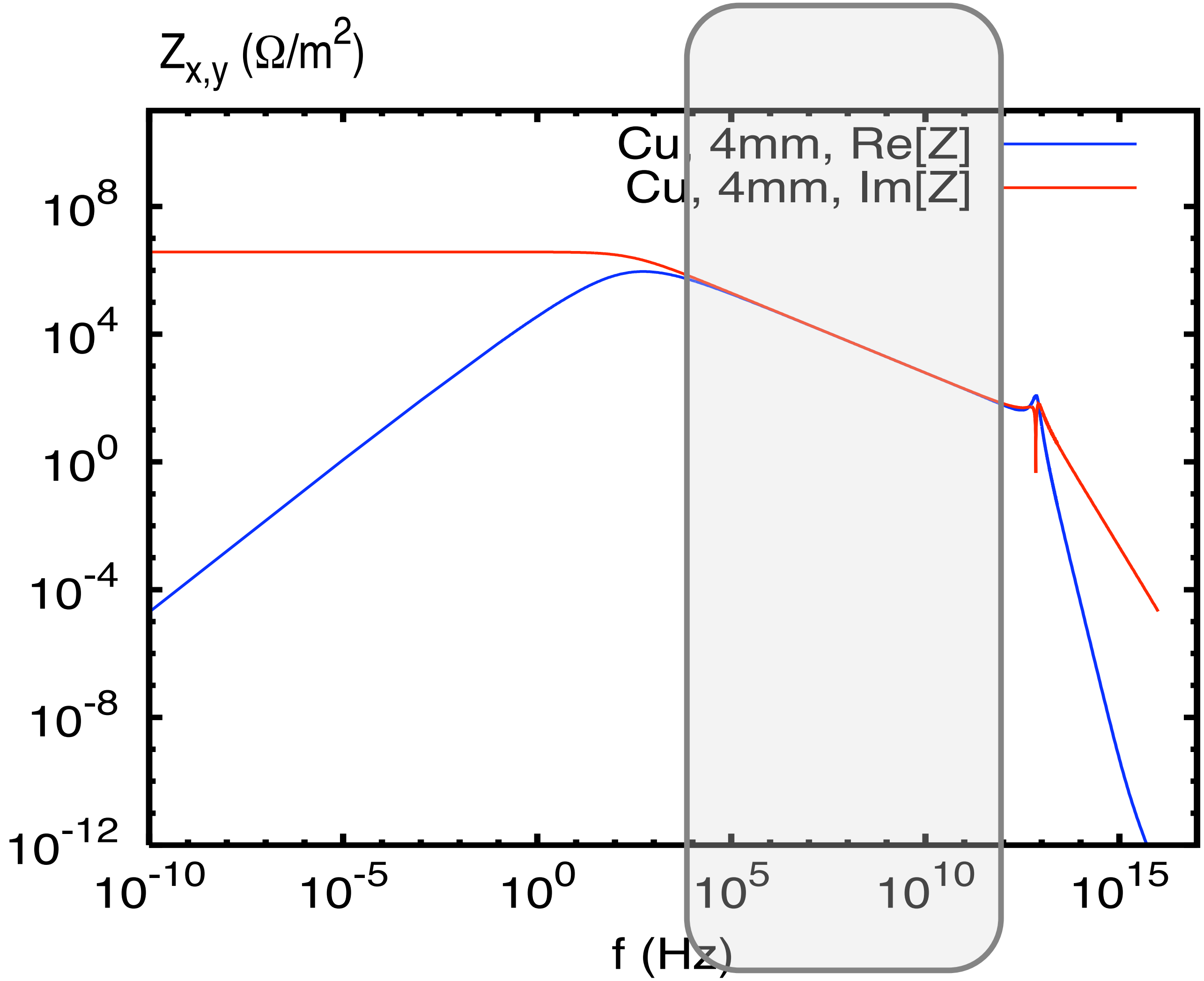


Examples of wakes/impedances
Resistive wall of beam chamber

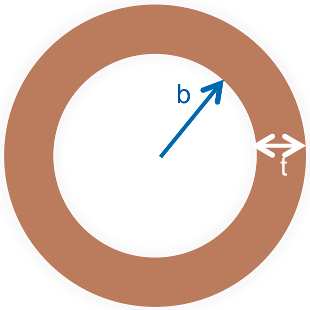


3 frequency regions of interest:

2. Between 10 kHz and 1 THz, the EM field is fully attenuated in the Cu layer and the impedance is like the one calculated assuming infinitely thick wall

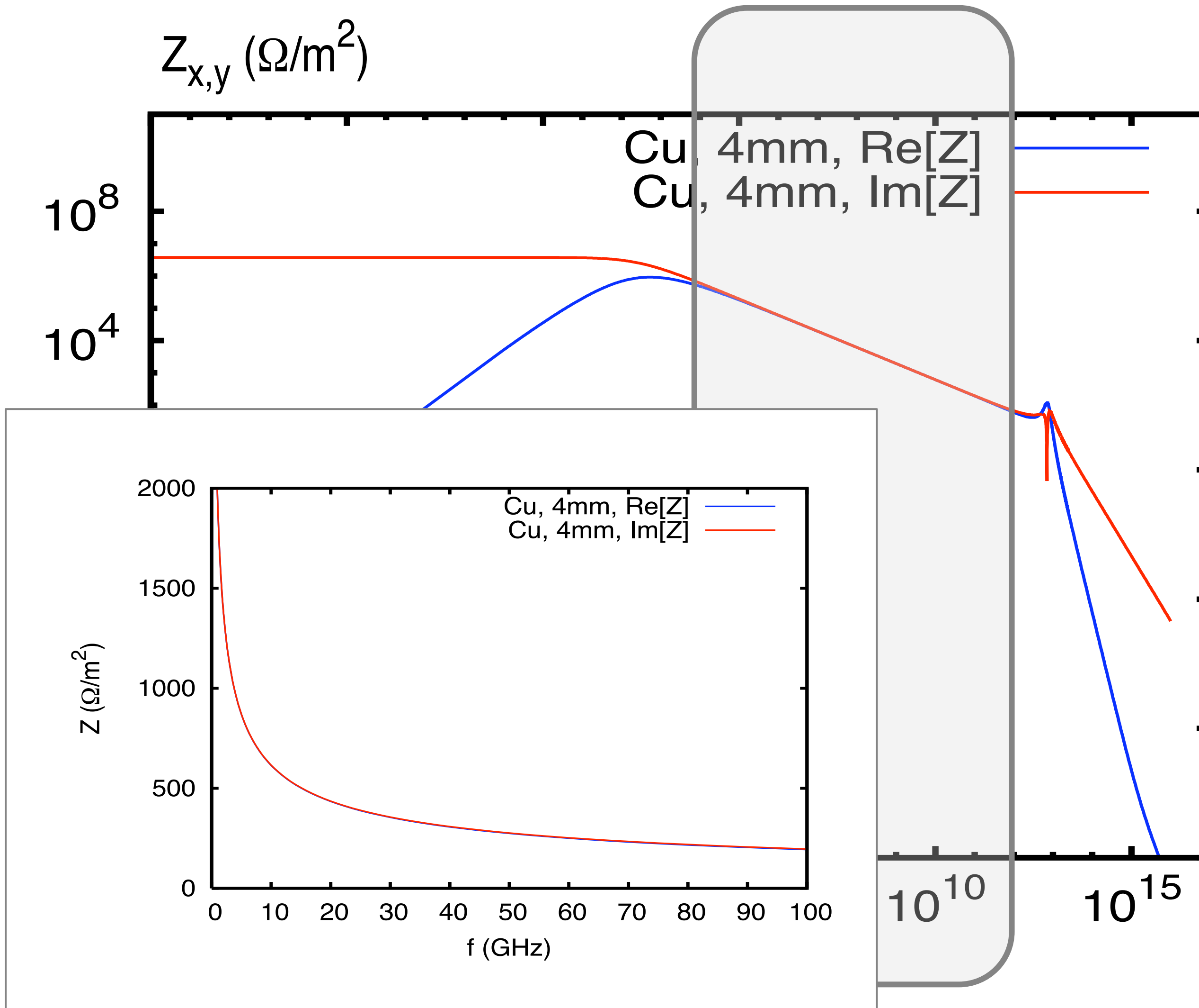


Examples of wakes/impedances
Resistive wall of beam chamber

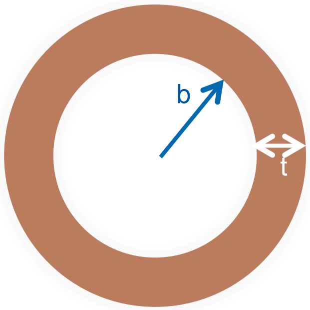


3 frequency regions of interest:

2. Between 10 kHz and 1 THz, the EM field is fully attenuated in the Cu layer and the impedance is like the one calculated assuming infinitely thick wall

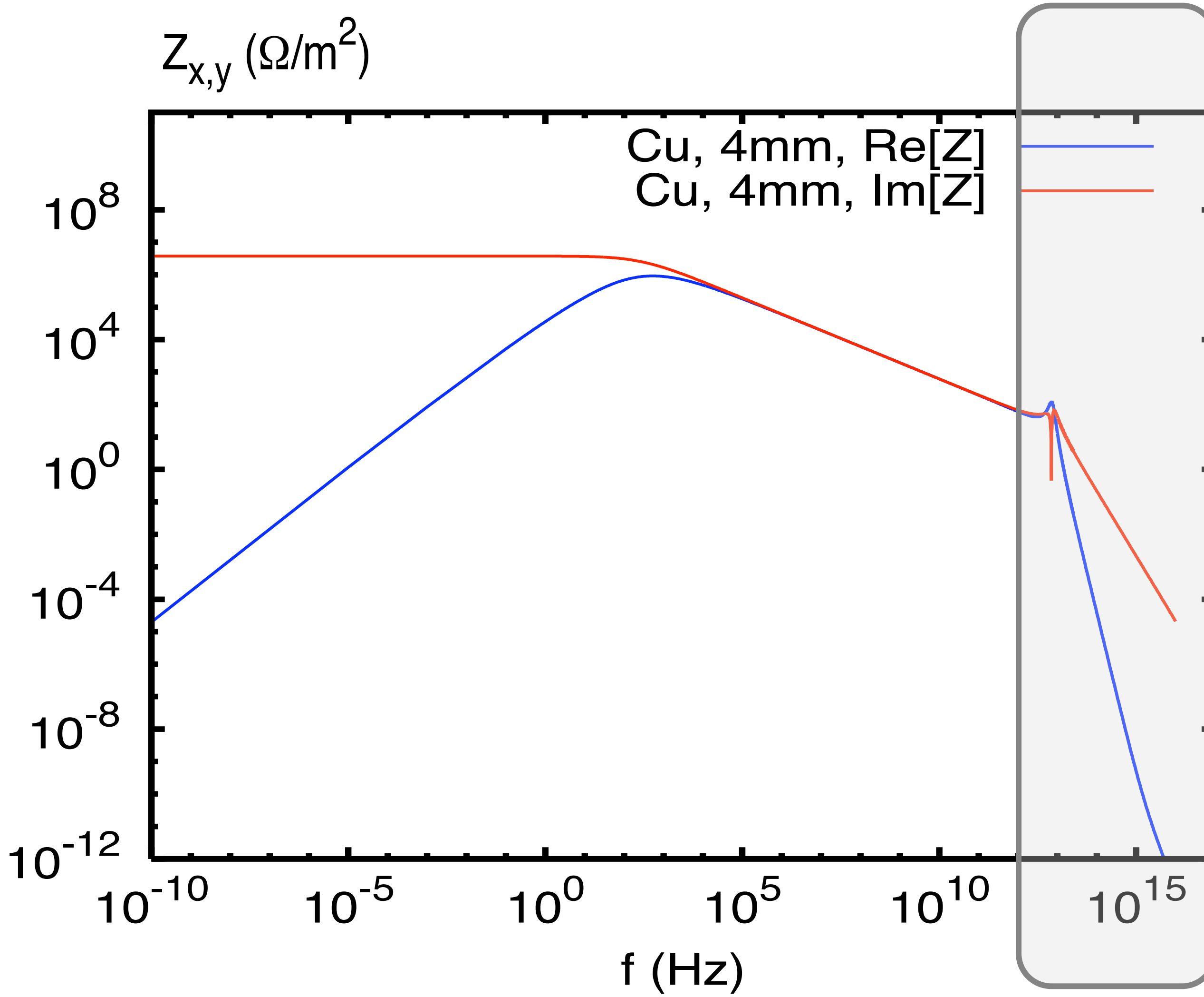


Examples of wakes/impedances
Resistive wall of beam chamber

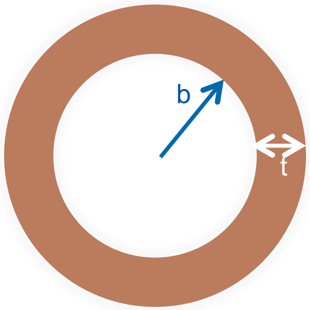


3 frequency regions of interest:

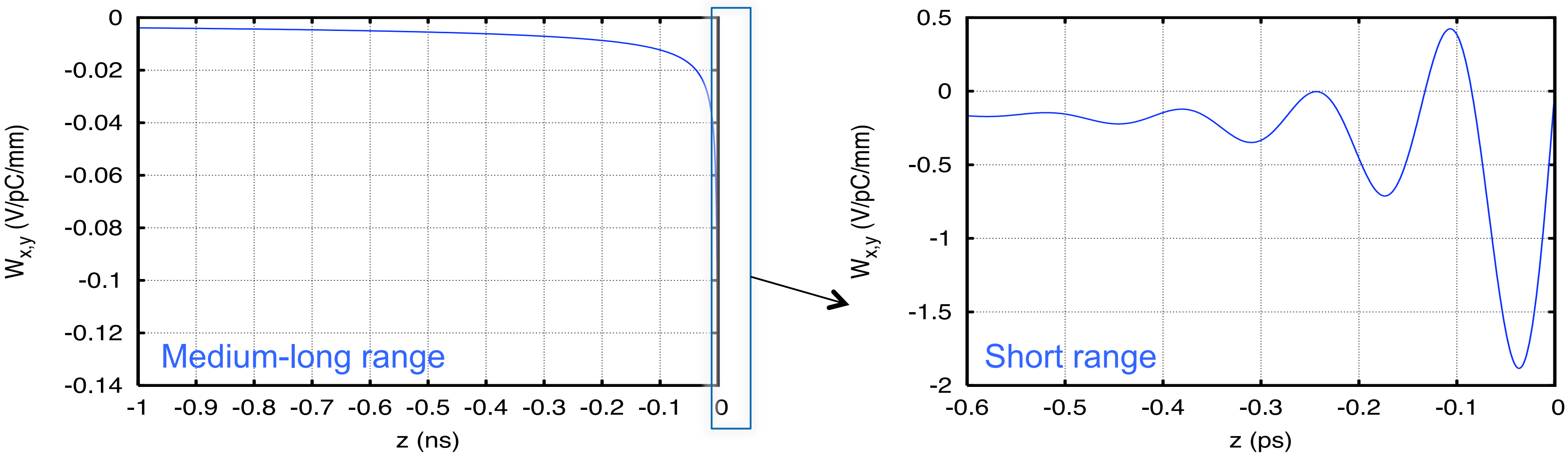
3. Above 1 THz, there is a resonance (100 THz region). In this region also ac conductivity should be taken into account



Examples of wakes/impedances
Resistive wall of beam chamber



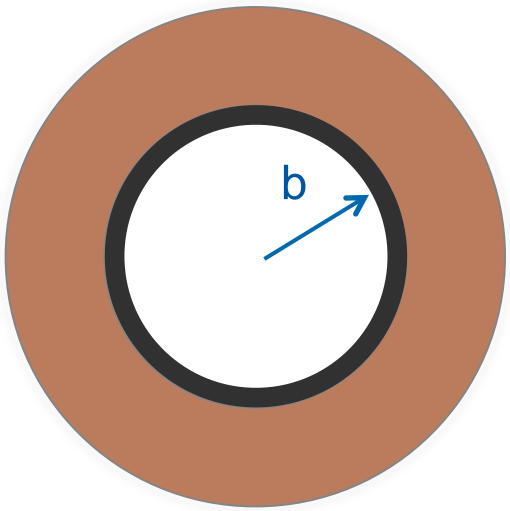
Correspondingly, in time domain, the wake exhibits different behaviours in short and long range



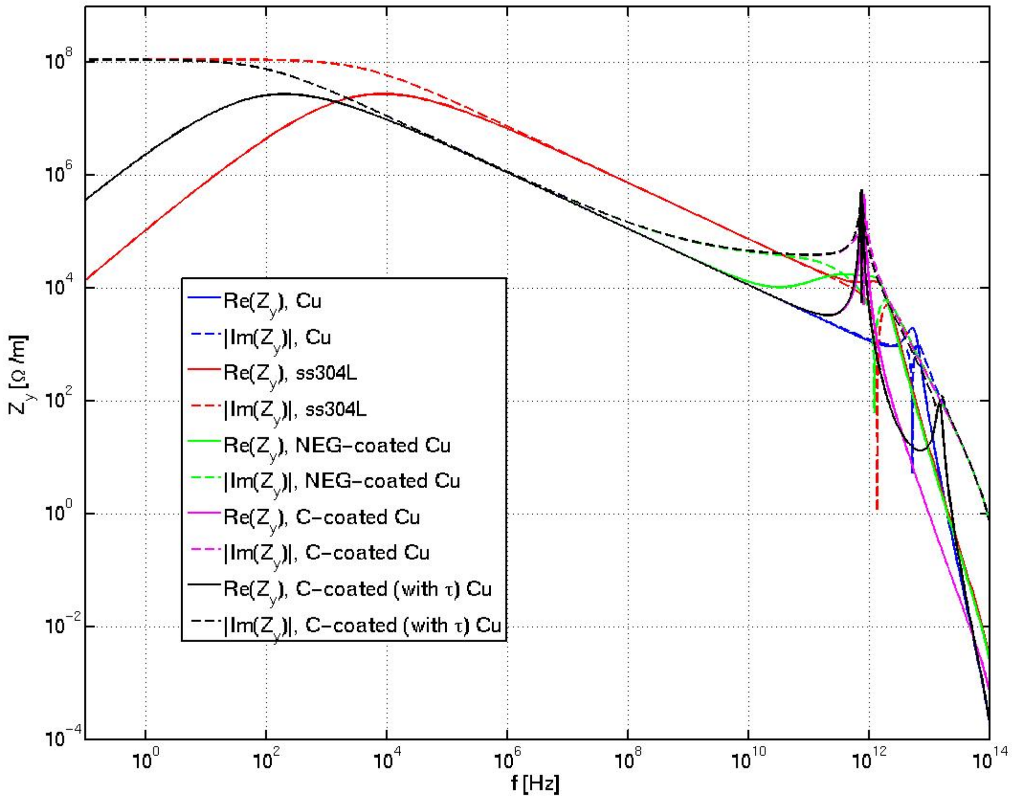
In the range of tenths of ns up to fractions of ms (e.g. bunch length to several turns for the SPS) monotonically decaying wake

From behind the source to ~1ps the wake has an oscillatory behaviour, associated to the high frequency resonance

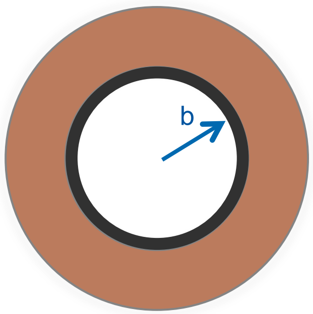
Examples of wakes/impedances
Resistive wall of beam chamber



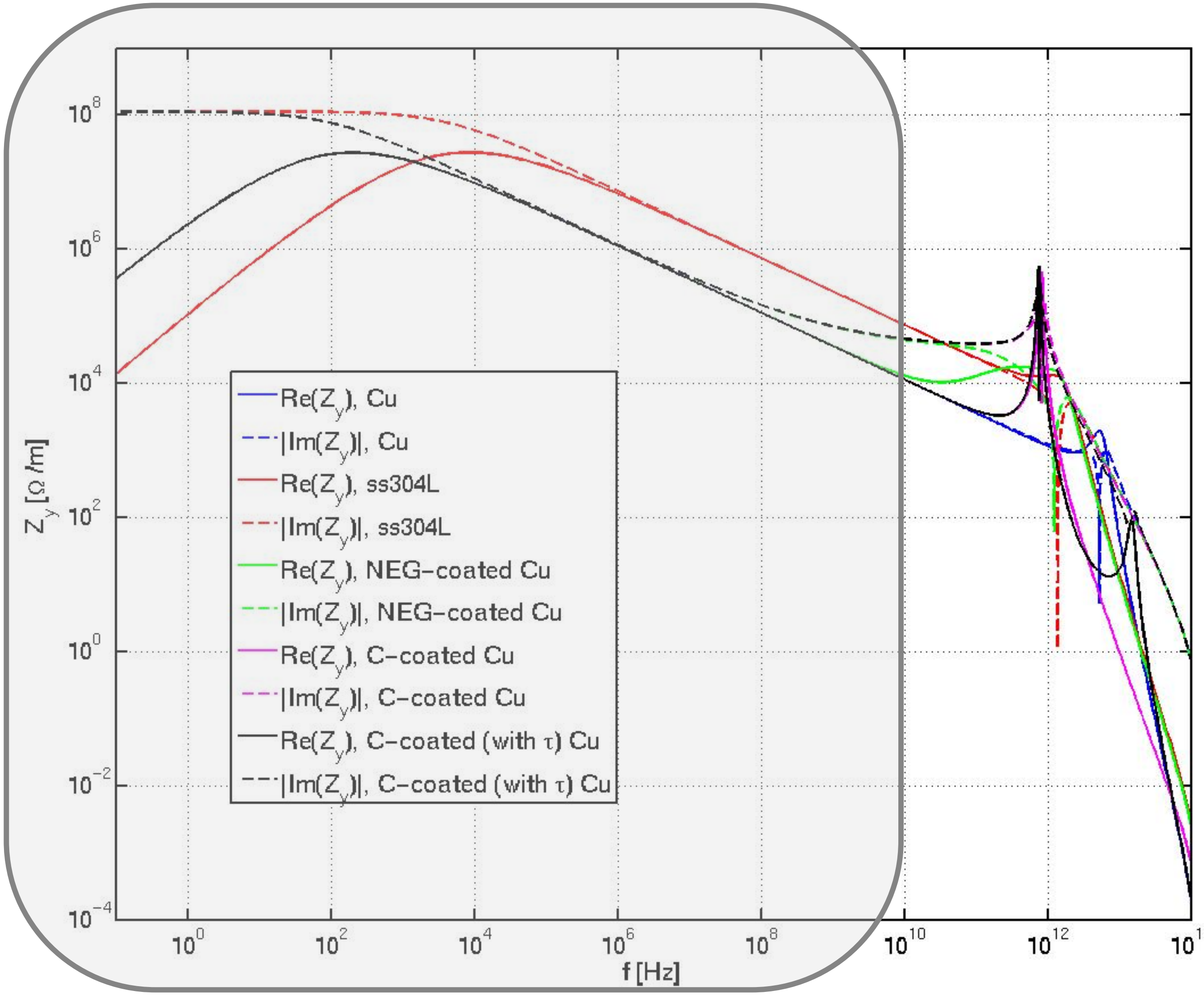
– Another interesting example: 1 m long Cu (or StSt) pipe with radius b=9 mm coated with a layer of thickness t = 1μm of NEG or amorphous carbon (a-C)



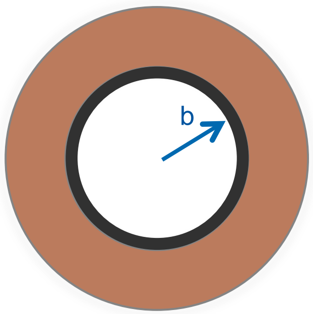
Examples of wakes/impedances
Resistive wall of beam chamber



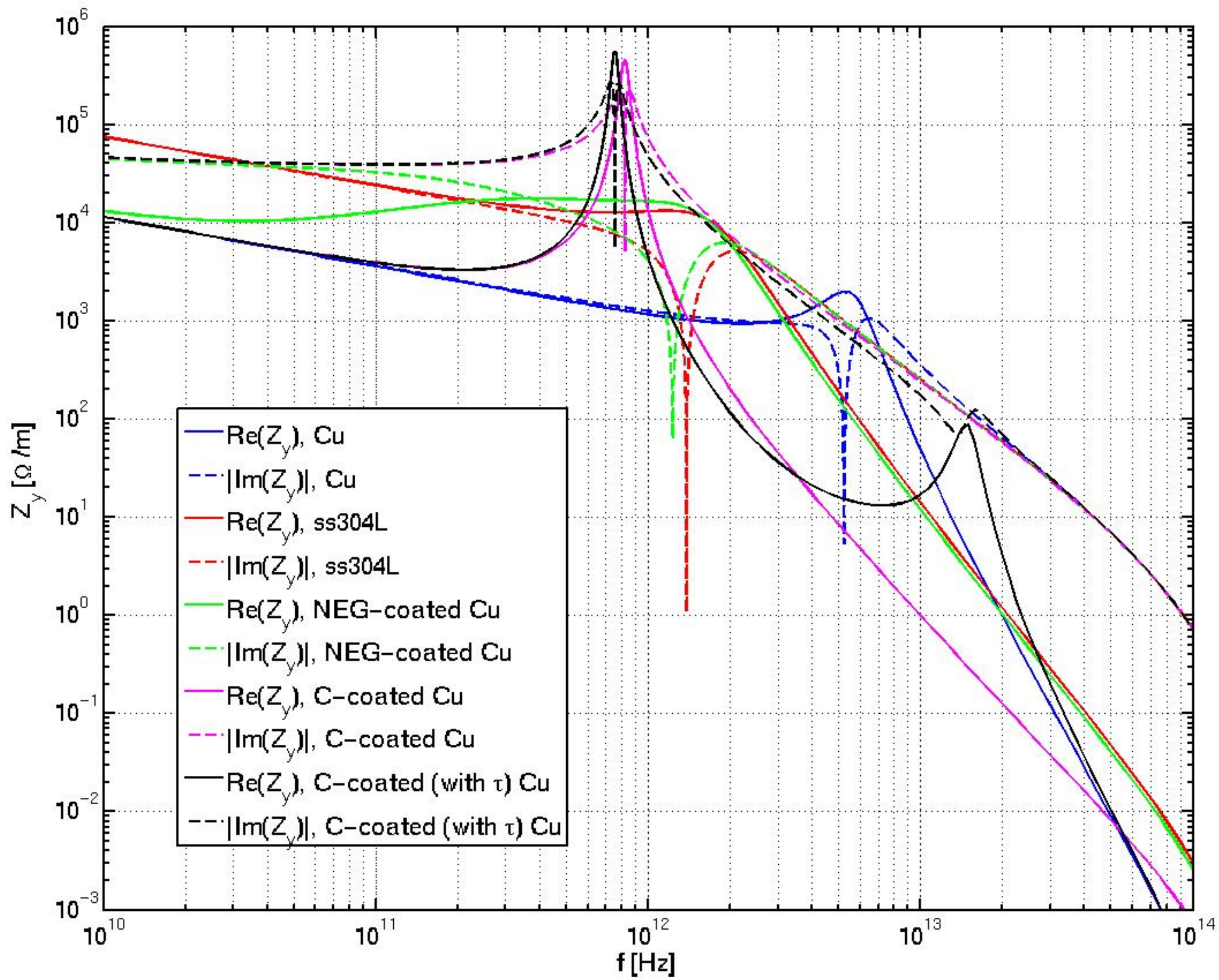
1. Below 10 GHz, coating is transparent. Impedance of StSt is about 7 times larger than that of Cu due to its conductivity about 50 times lower



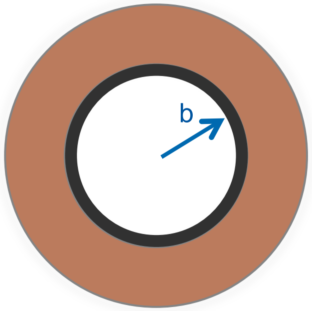
Examples of wakes/impedances Resistive wall of beam chamber



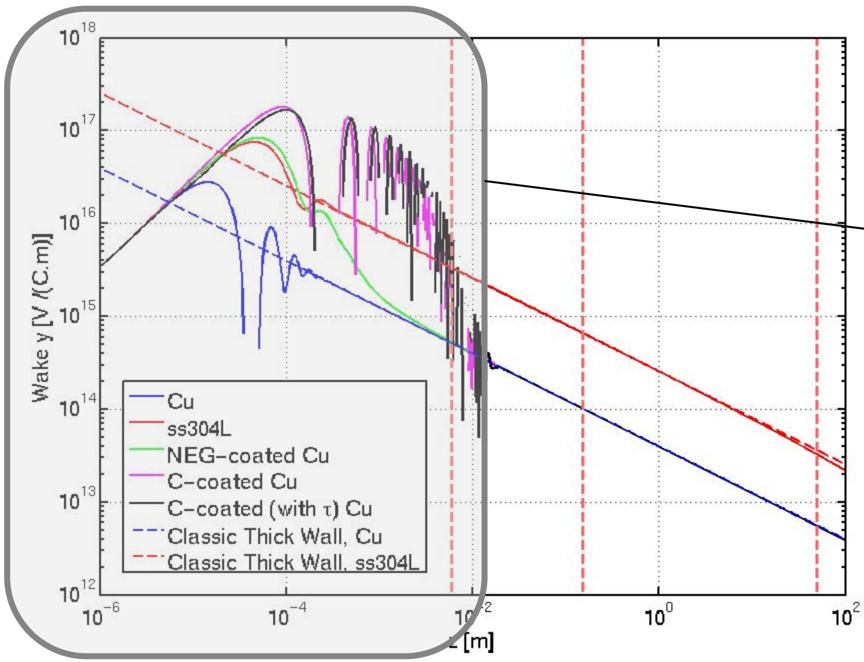
2. Peak of NEG coated Cu is about the same as StSt (NEG and StSt have about same value of conductivity)
3. a-C causes the Cu peak to lower frequency and makes it more pronounced.
4. If ac conductivity included in a-C there is another peak at higher frequency

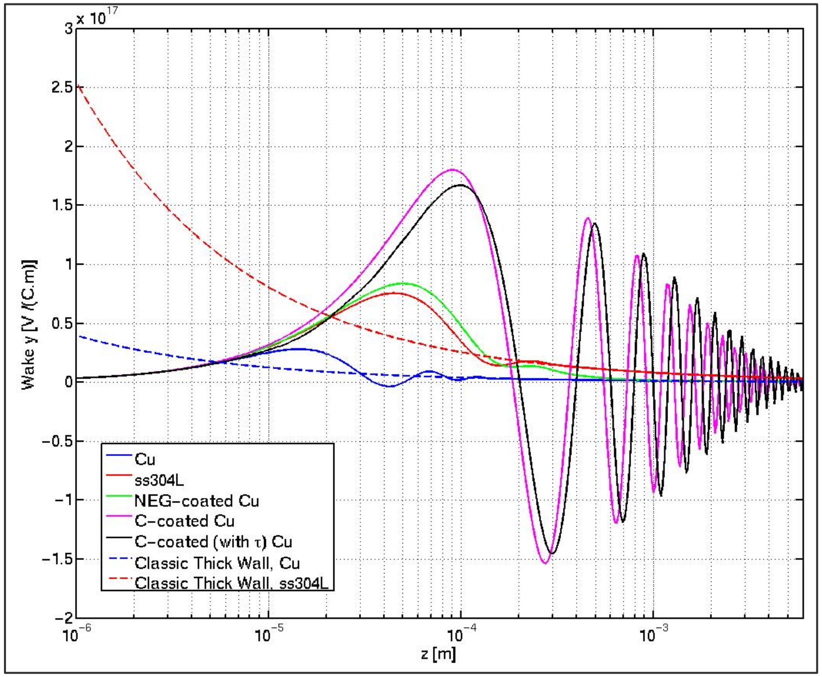


Examples of wakes/impedances
Resistive wall of beam chamber

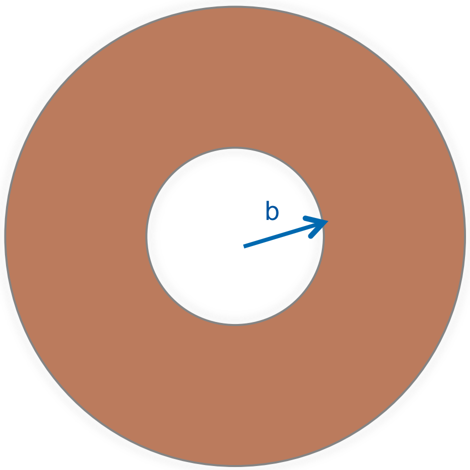


Correspondingly, in time domain, the wake exhibits different behaviours for the different cases only in the short range (plots with z positive to use the log scale)





Examples of wakes/impedances
Equations for infinitely thick wall



$$\frac{W_{RW||}(z)}{L} = -\frac{c}{4\pi b}\sqrt{\frac{Z_0}{\pi\sigma|z|^3}}$$

$$\frac{W_{RW(x,y)}(z)}{L} = \frac{c}{\pi b^3}\sqrt{\frac{Z_0}{\pi\sigma|z|}}$$

valid only in the range

$$b\chi^{1/3} \ll |z| \ll \frac{b}{\chi}$$

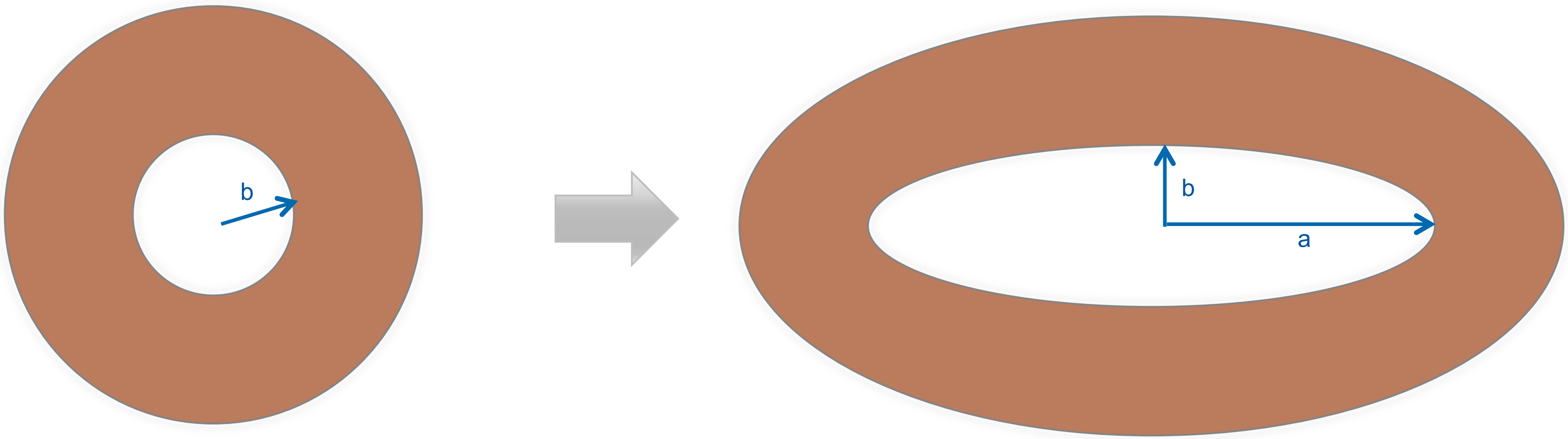
with $\chi = \frac{1}{Z_0\sigma b}$

$$\frac{Z_{RW||}(\omega)}{L} = \frac{1}{4\pi b}\sqrt{\frac{2Z_0|\omega|}{\sigma c}}\left[1+\mathrm{sgn}(\omega)\cdot i\right]$$

$$\frac{Z_{RW(x,y)}(\omega)}{L} = \frac{1}{2\pi b^3}\sqrt{\frac{2Z_0 c}{\sigma|\omega|}}\left[1+\mathrm{sgn}(\omega)\cdot i\right]$$

valid in the corresponding range of frequencies

Examples of wakes/impedances
Equations for infinitely thick wall



$$W^{\text{ell}}_{RW||}(z;a,b) = Y^{longitudinal}(a,b)W_{RW}(z;b)$$

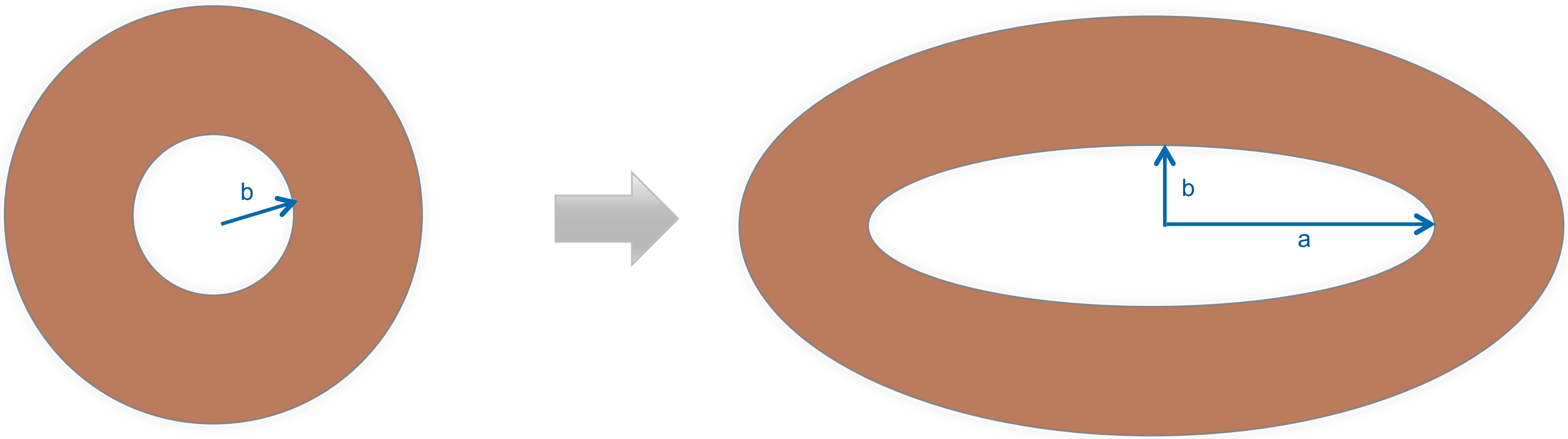
$$W^{\text{ell}}_{RWx}(z;a,b) = Y^{dipolar}_{x}(a,b)W_{RW}(z;b)$$

$$W^{\text{ell}}_{RWQx}(z;a,b) = Y^{quadrupolar}_{x}(a,b)W_{RW}(z;b)$$

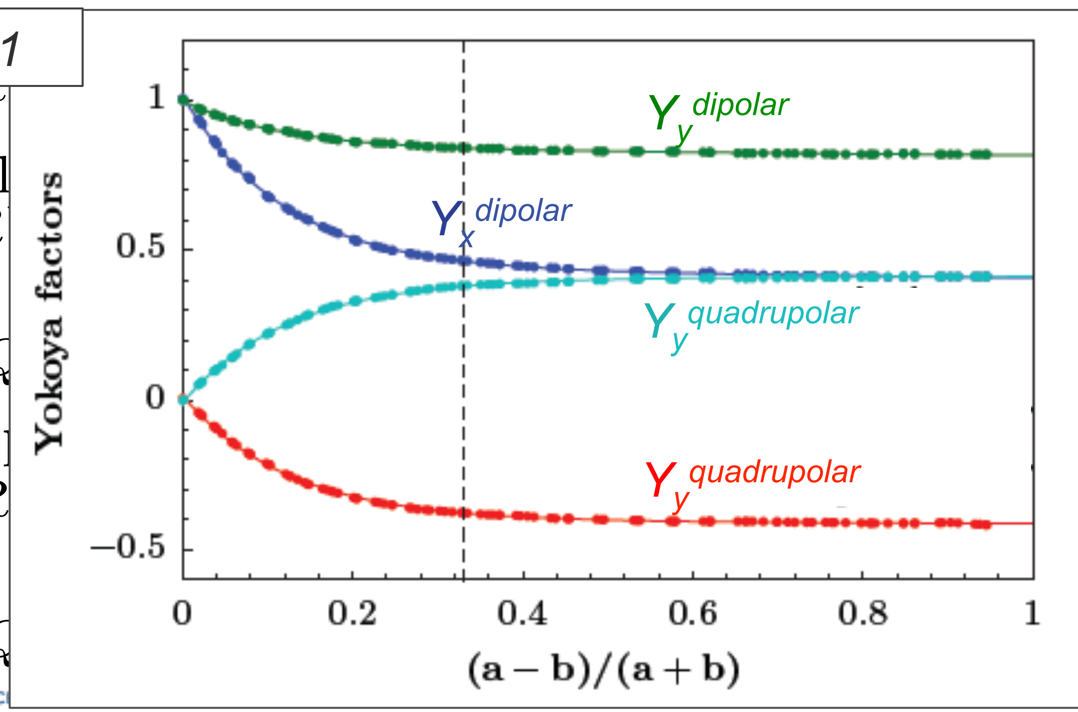
$$W^{\text{ell}}_{RWy}(z;a,b) = Y^{dipolar}_{y}(a,b)W_{RW}(z;b)$$

$$W^{\text{ell}}_{RWQy}(z;a,b) = Y^{quadrupolar}_{y}(a,b)W_{RW}(z;b)$$

Examples of wakes/impedances
Equations for infinitely thick wall

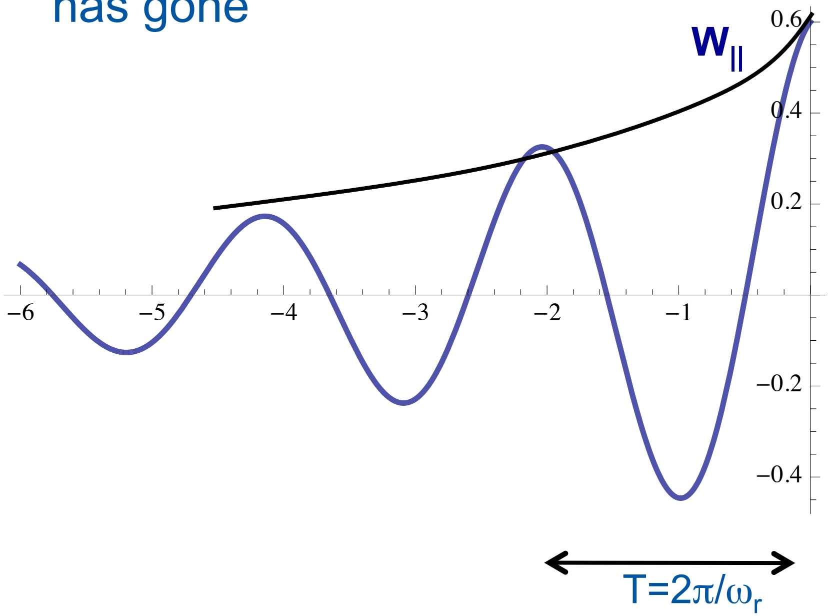


$Y^{longitudinal} \approx 1$

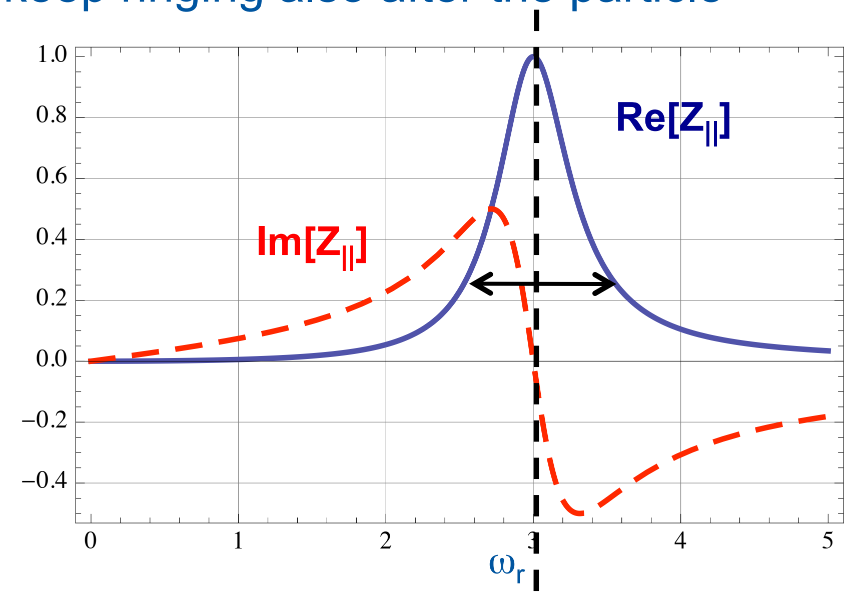


Examples of wakes/impedances
Longitudinal narrow band resonator

All cavity-like objects behave like one or more narrow band resonators, as a charged particle is likely to excite modes that keep ringing also after the particle has gone

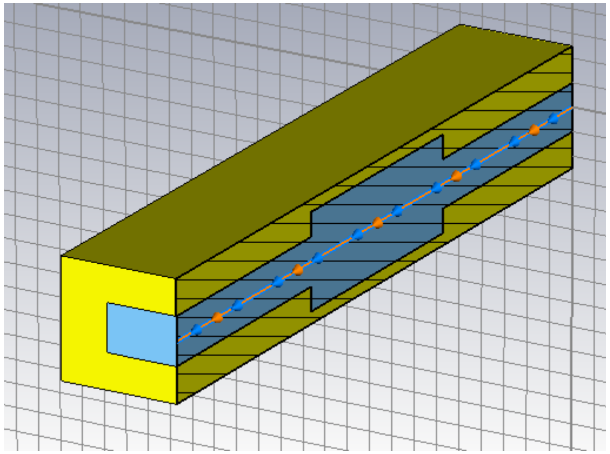


$\mathcal{F}$ $\Longleftrightarrow$



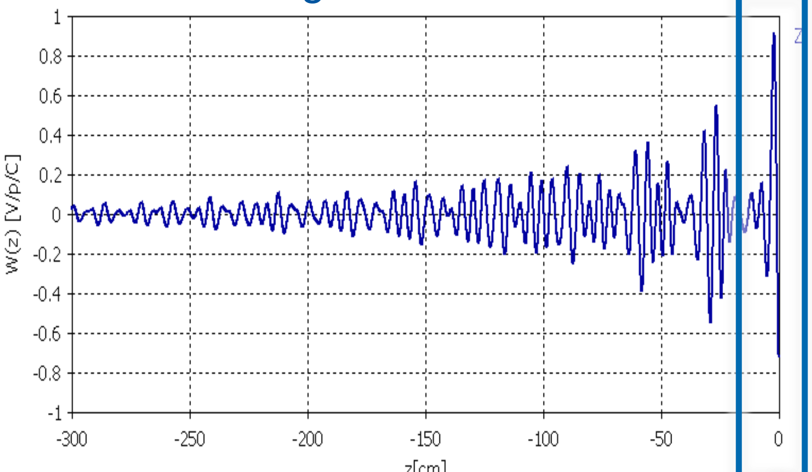
- The frequency ω_r is related to the oscillation of E_z, and therefore to the frequency of the mode excited in the object
- The decay time depends on whether modes are excited and how quickly the stored energy in these modes is dissipated (quantified by a quality factor Q)

Examples of wakes/impedances
Single cell cavity

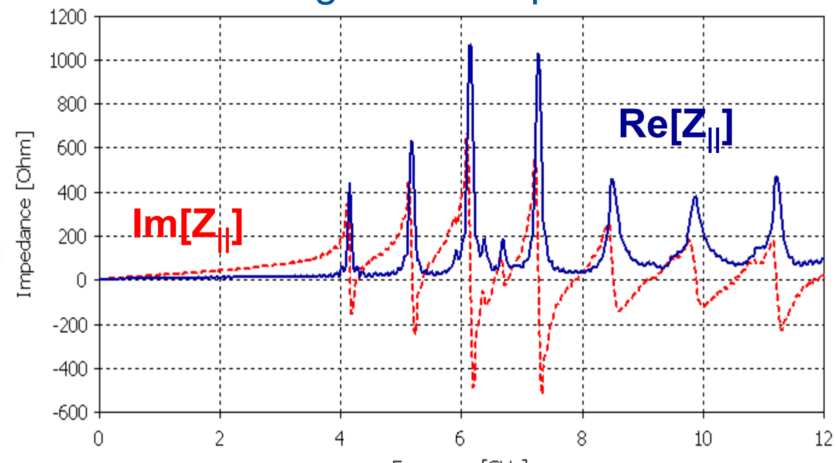


- A more complex example: a simple pill-box cavity with walls having finite conductivity
- Several modes can be excited
  - Below the pipe cut-off frequency the width of the peaks is only determined by the finite conductivity of the walls
  - Above, losses also come from propagation in the chamber

Longitudinal wake

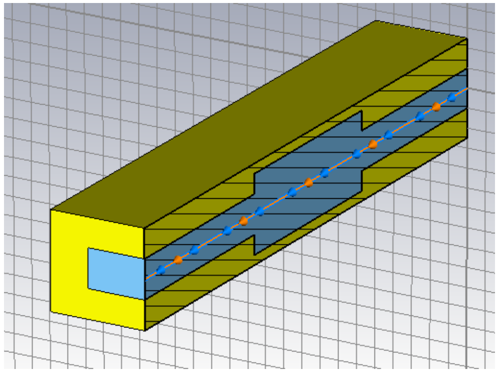


Longitudinal impedance

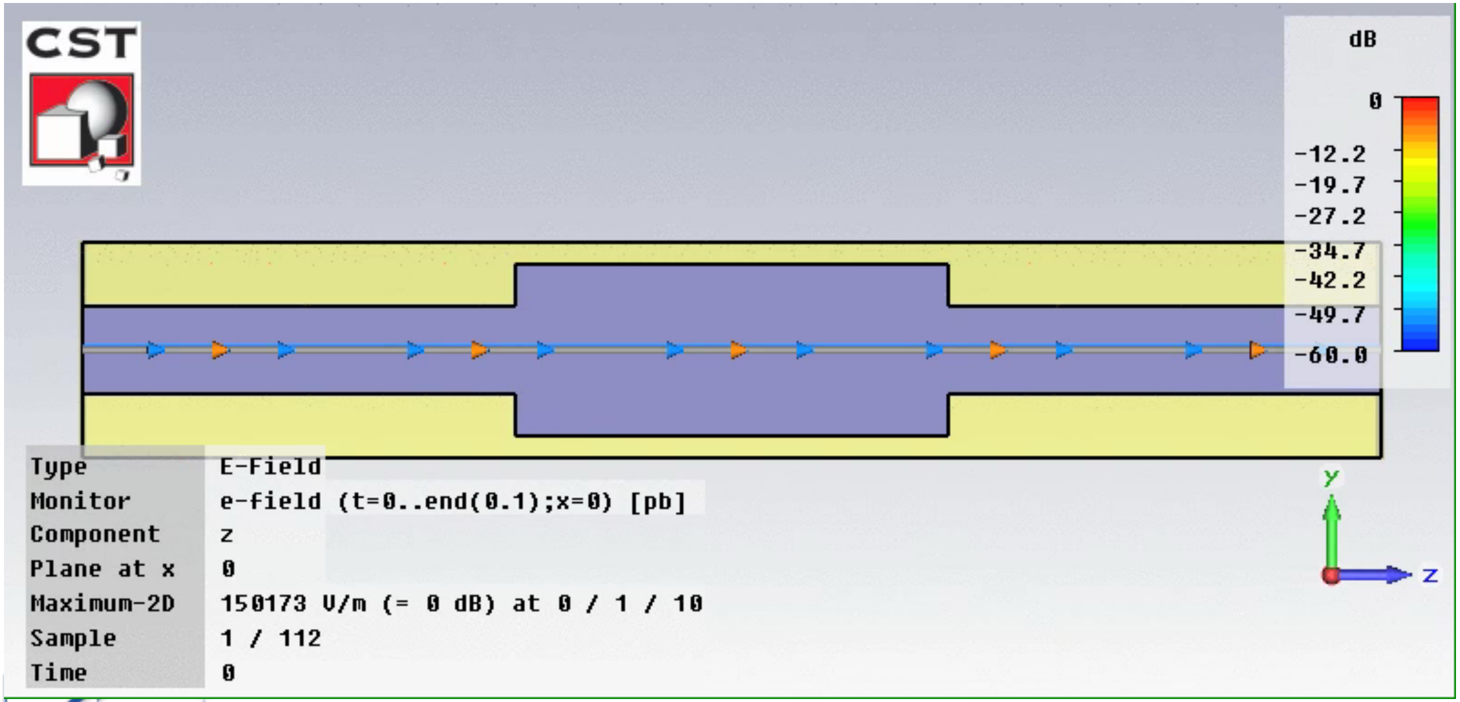


Note $W_{||}(0)$<0, **CST** uses opposite convention!!

Examples of wakes/impedances
Single cell cavity

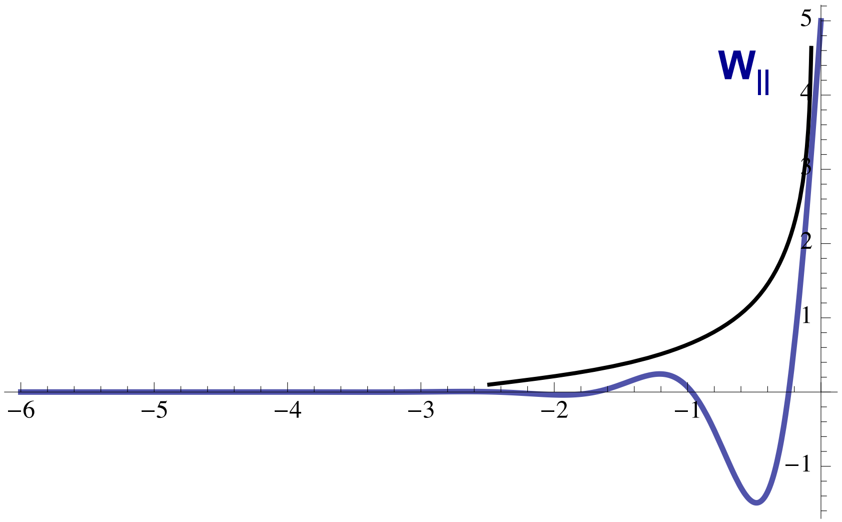


– Evolution of the the longitudinal electric field (E_z) in the cavity while and after the beam has passed

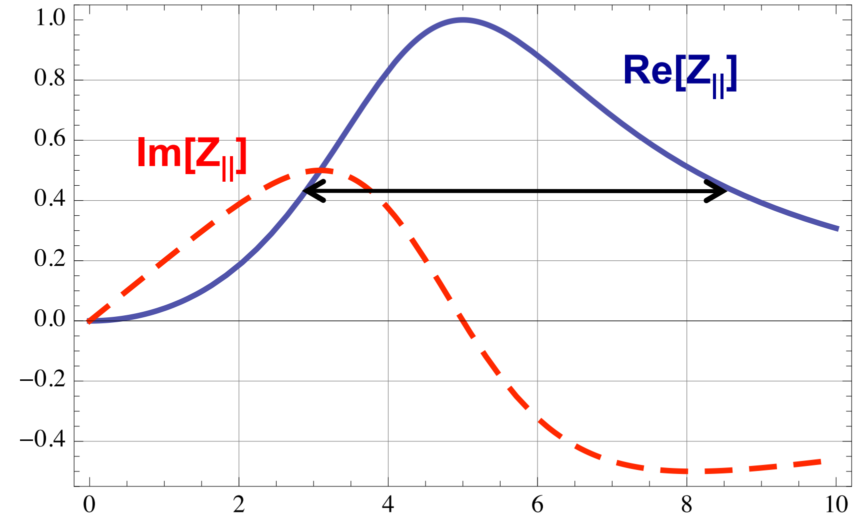


Examples of wakes/impedances
Longitudinal broad band resonator

Objects whose geometry makes it unlikely for a charged particle to excite modes exhibit fast decaying wakes, associated to a broad frequency coverage

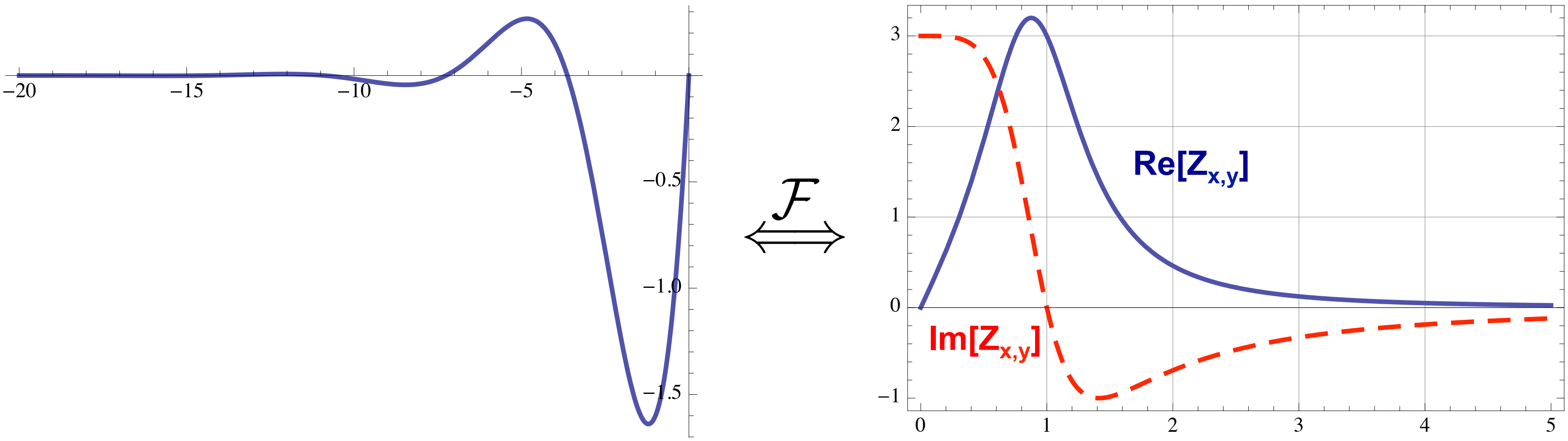


$\overset{\mathcal{F}}{\Longleftrightarrow}$



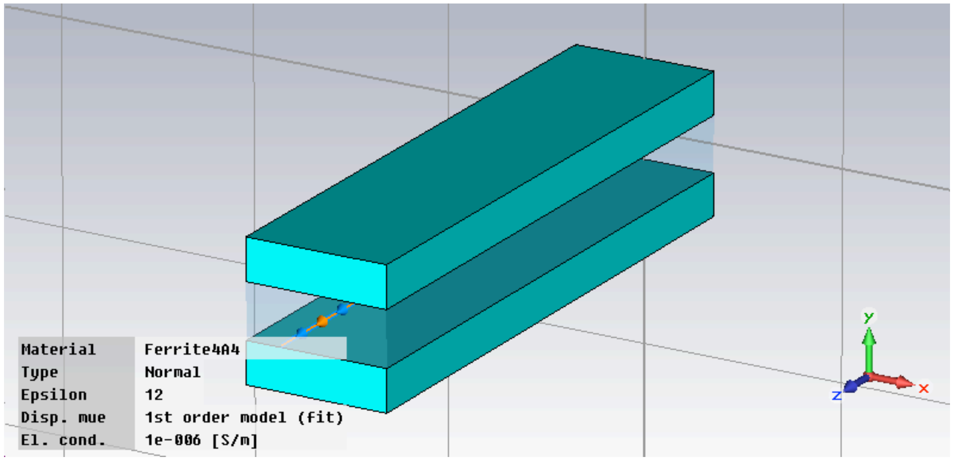
- In most cases, impedances of devices can be modeled as the sum of several narrow- and broad-band resonator peaks
- Other contributors to the global impedance can also have different shapes, e.g. the resistive wall

Examples of wakes/impedances
Transverse resonator

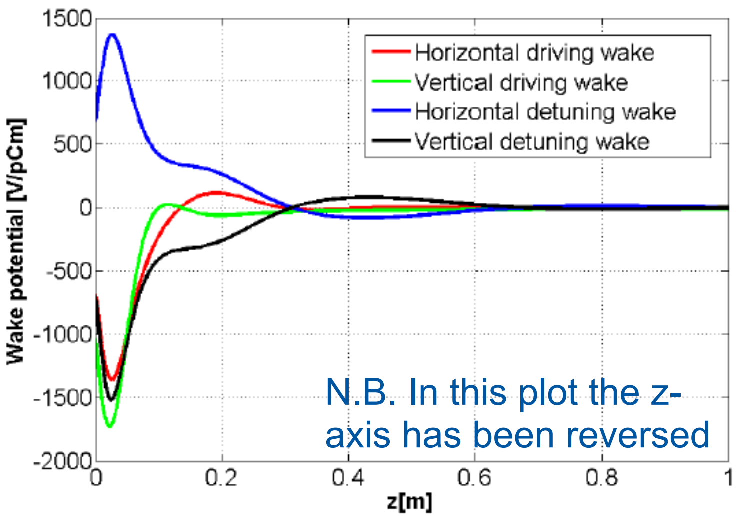


– Shape of wake function can be similar to that in longitudinal plane, determined by the oscillation period of the trailing electromagnetic fields
– Contrary to longitudinal impedances, Re[$Z_{x,y}$] is an odd function of frequency, while Im[$Z_{x,y}$] is an even function

Examples of wakes/impedances
Ferrite kicker: simple model



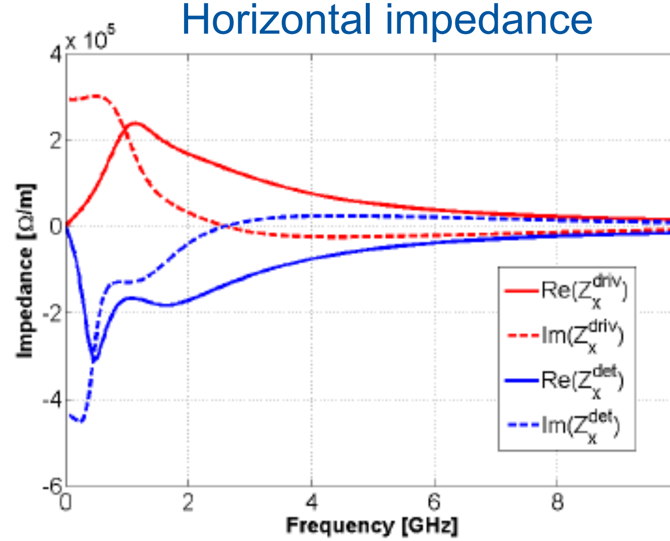
- An example: magnetic kickers are usually large contributors to the transverse impedance of a machine
- It is a broad band contribution
  - No trapped modes
  - Losses both in vacuum chamber and ferrite (kicker heating and outgassing)



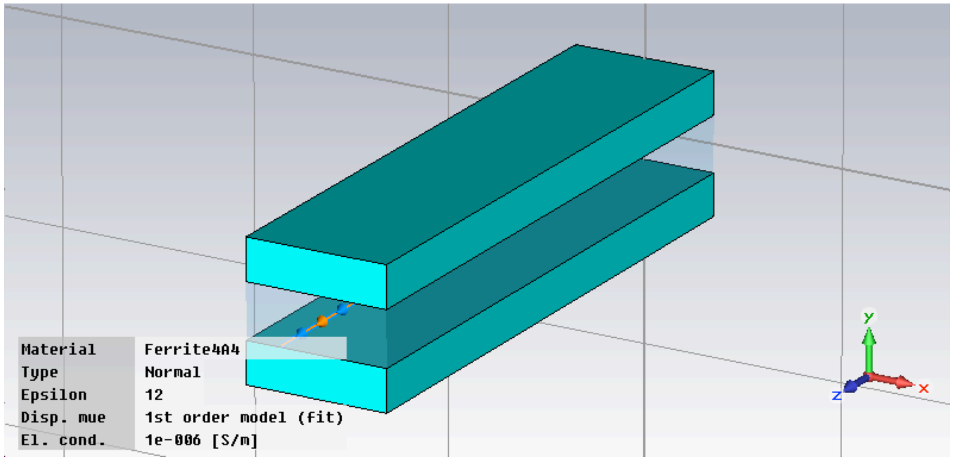
N.B. In this plot the z-axis has been reversed

$\mathcal{F}$ $\Longleftrightarrow$

Horizontal impedance



Examples of wakes/impedances
Ferrite kicker: simple model



- An example: magnetic kickers are usually large contributors to the transverse impedance of a machine
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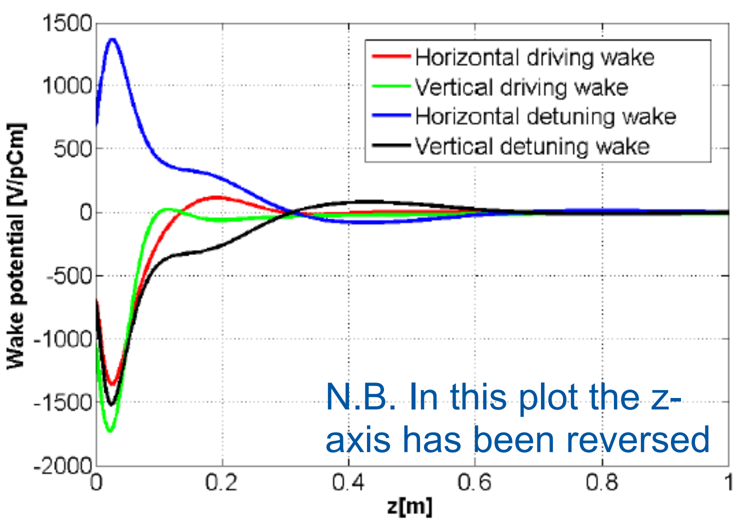
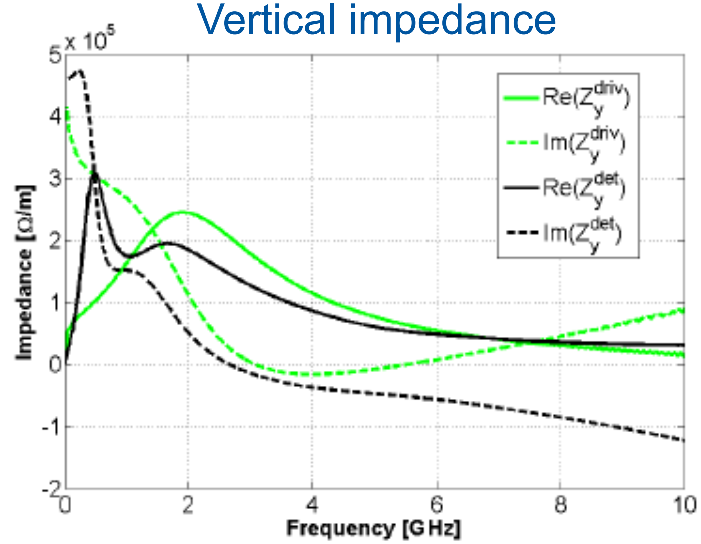


N.B. In this plot the z-axis has been reversed

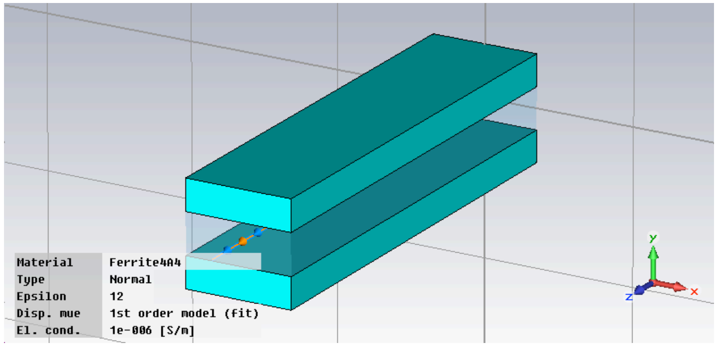
$\overset{\mathcal{F}}{\Longleftrightarrow}$

Vertical impedance

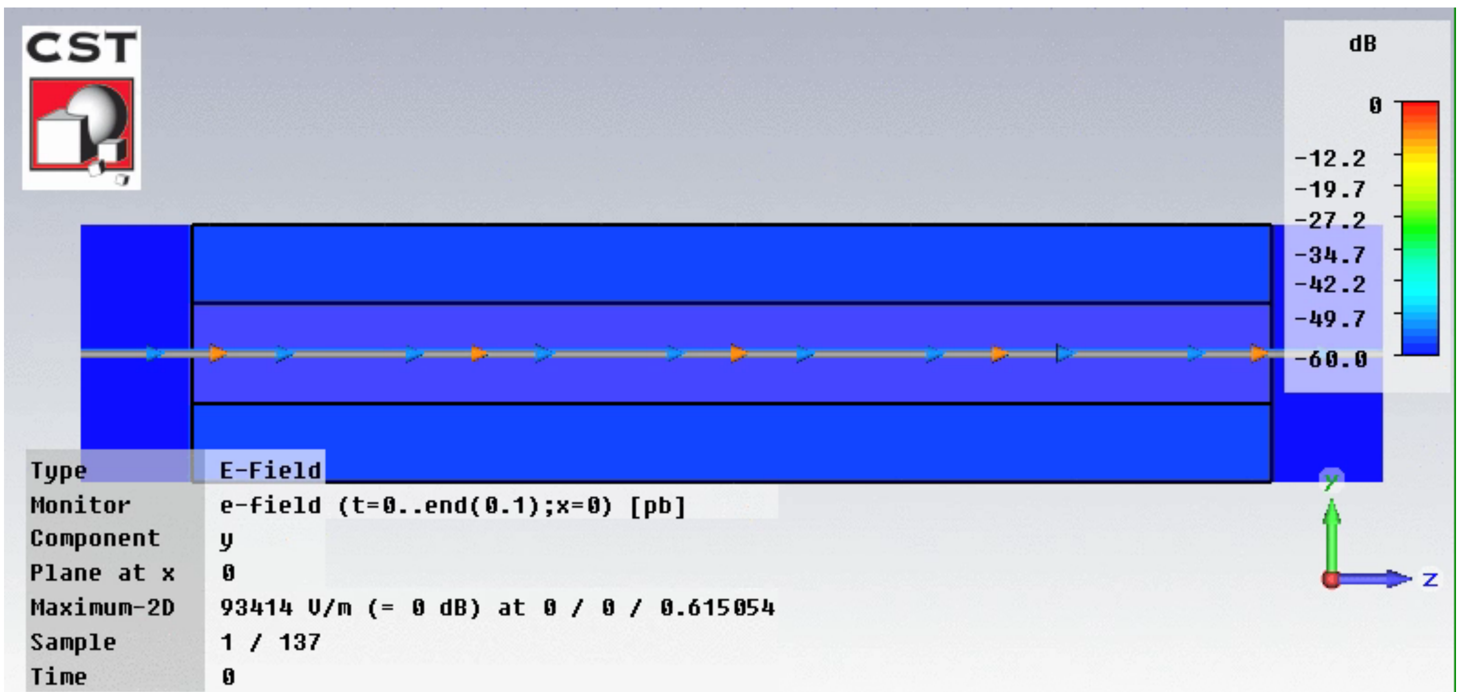


In the EM simulation wakes do not go to zero when z=0 due to finite size of excitation

Examples of wakes/impedances
Ferrite kicker: simple model



– Evolution of the the vertical electric field (E_y) in the kicker while and after the beam has passed

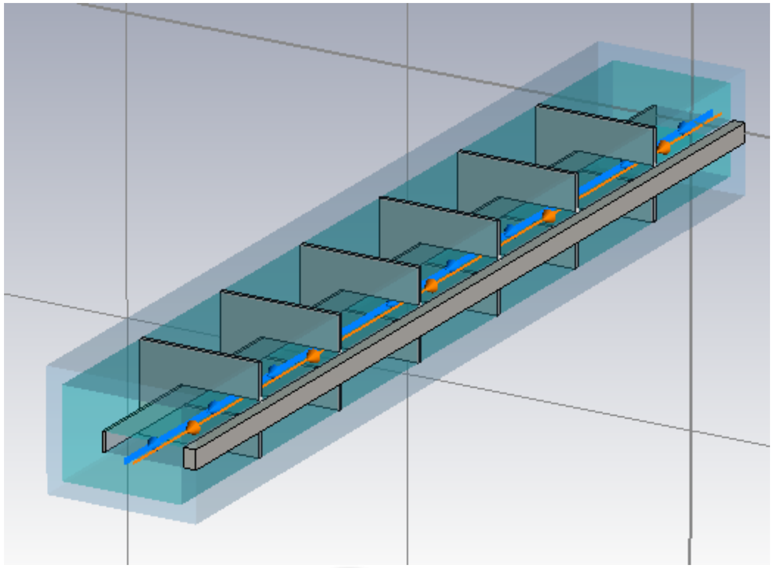


Examples of wakes/impedances
The SPS kickers

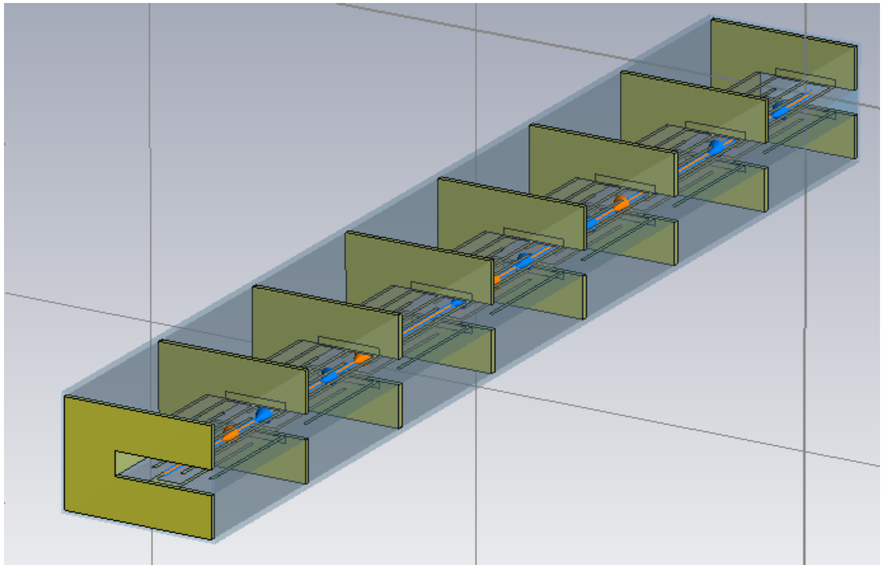
Two types of SPS extraction kickers (MKE):

1. Original design: several modules separated by conductor stripes (segmentation) with bare ferrite blocks, fed by an inner and an outer conductor
2. New design: like original, but modules have ‘serigraphed’ ferrite blocks (i.e. with patterns of silver paste screen printed on the ferrite surface exposed to the beam)

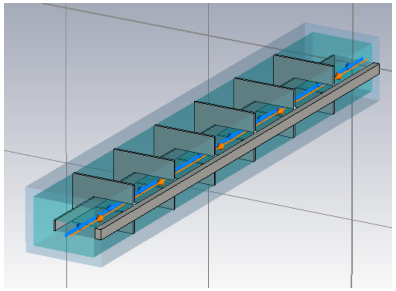
Original kicker



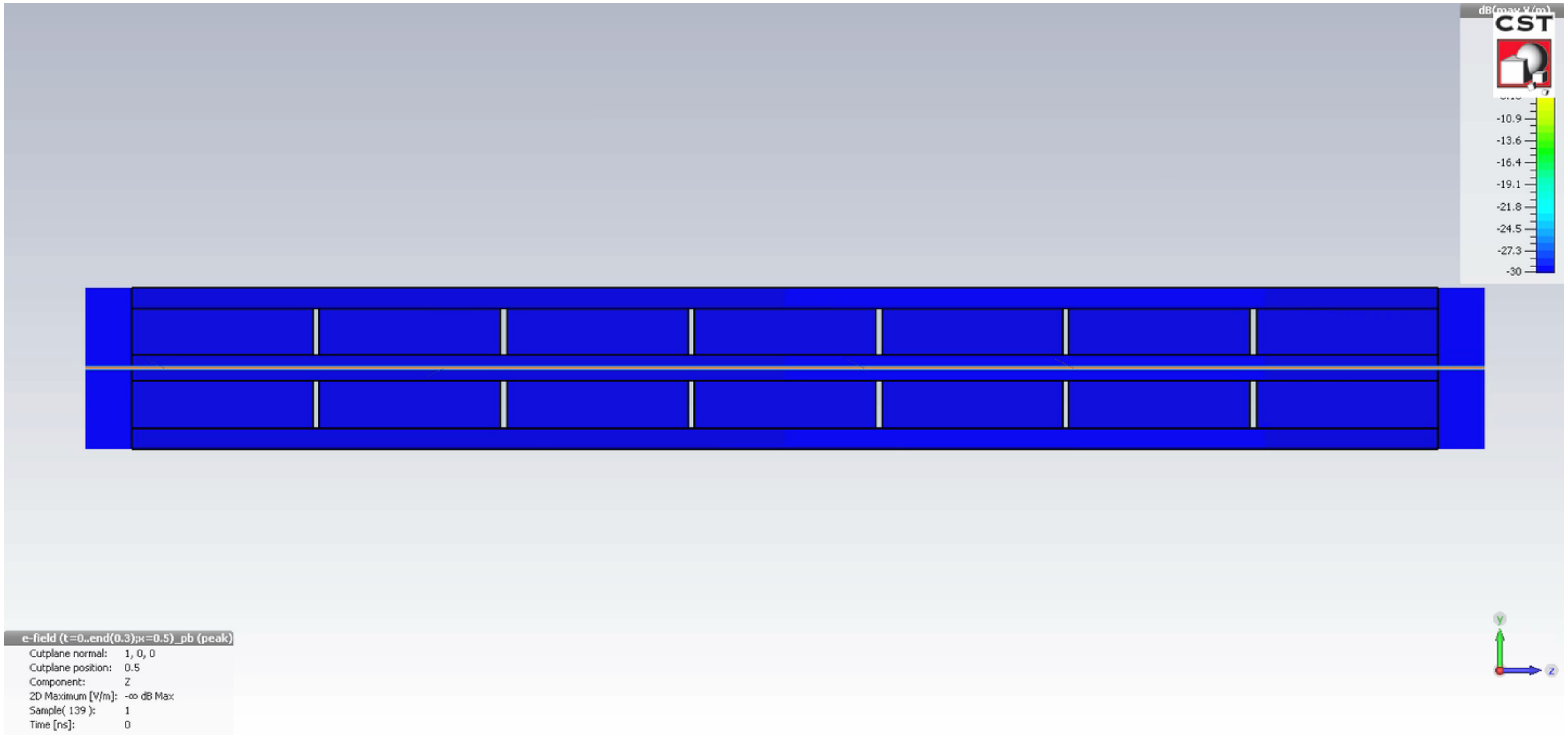
Serigraphed kicker



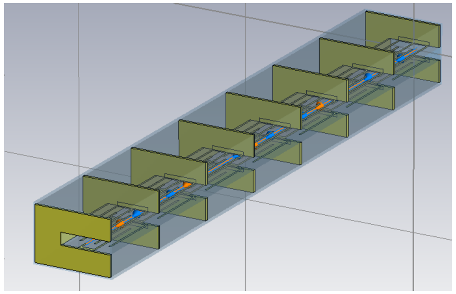
Examples of wakes/impedances
Ferrite kicker: simple model



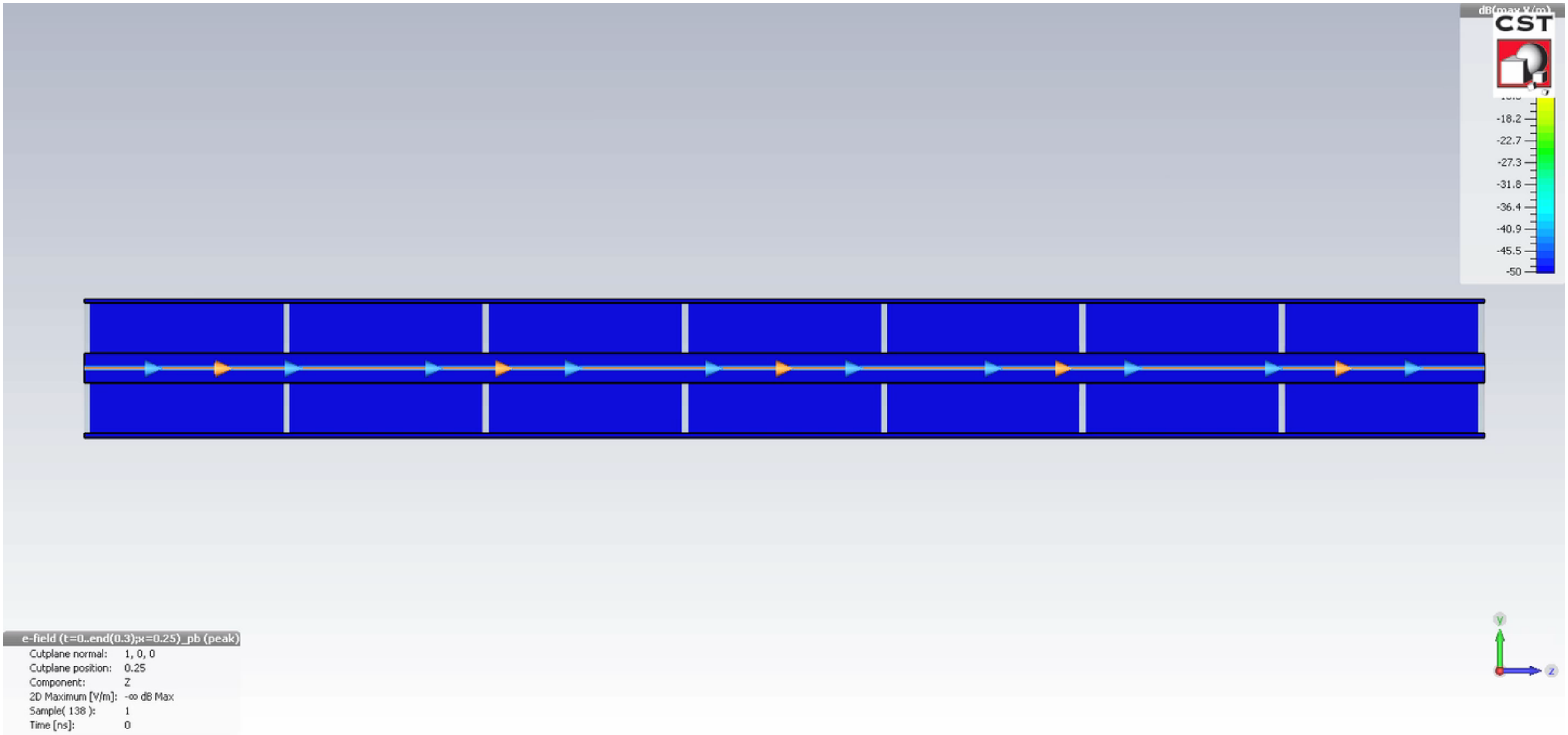
– Original kicker without serigraphy, typical broad-band behaviour, here some ringing is due to the longitudinal segmentation



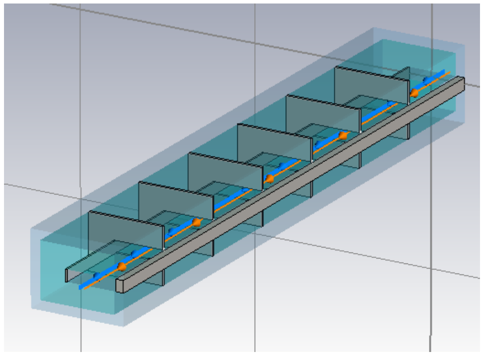
Examples of wakes/impedances Ferrite kicker: simple model

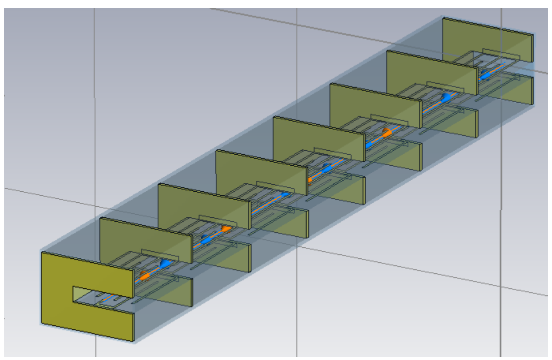


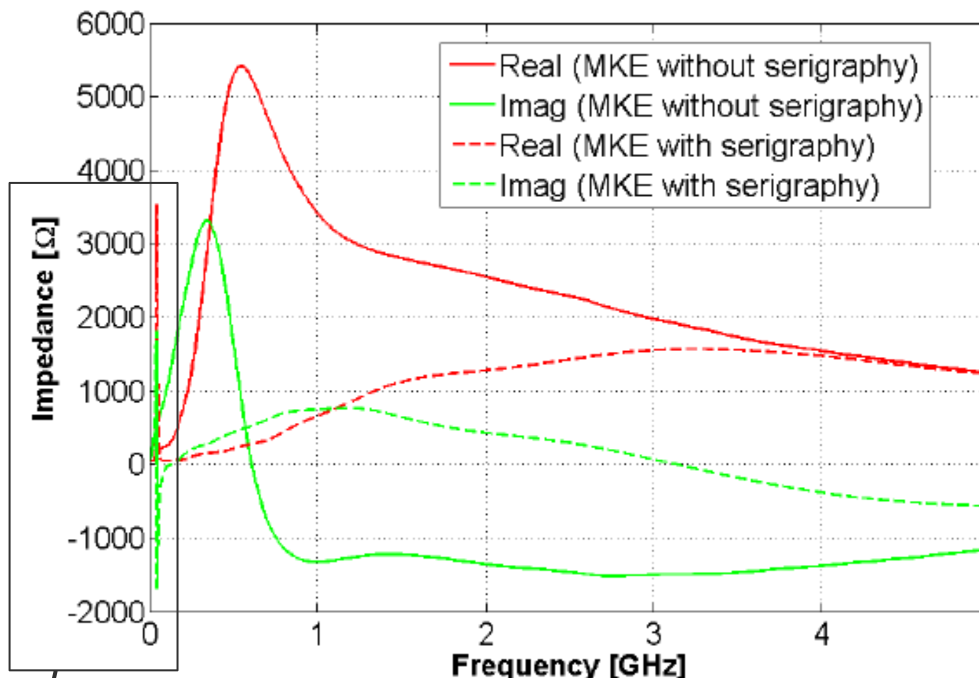
– Serigraphed kicker exhibits strong ringing due to the EM trapping along the serigraphy fingers

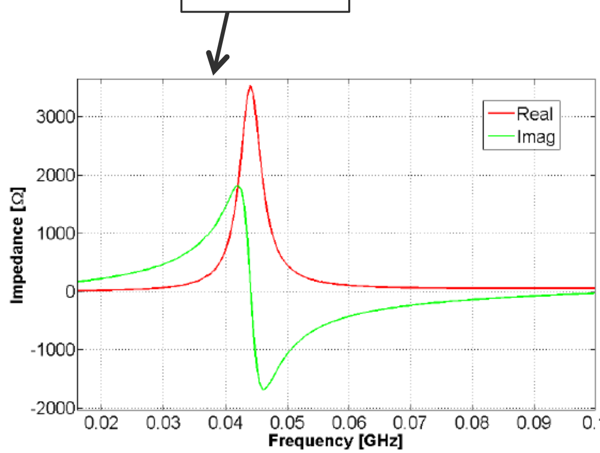


Examples of wakes/impedances
Ferrite kicker: simple model









Serigraphy

1. Reduces the broad-band contribution
2. Introduces resonance peak around 44 MHz

Examples of wakes/impedances
Equations of resonators

$$Z_{||}^{\mathrm{Res}}(\omega) = \frac{R_{s||}}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

$$W_{||}^{\mathrm{Res}}(z) = \begin{cases} 2\alpha_z R_{s||} \exp\left(\frac{\alpha_z z}{c}\right)\left[\cos\left(\frac{\bar{\omega} z}{c}\right) + \frac{\alpha_z}{\bar{\omega}} \sin\left(\frac{\bar{\omega} z}{c}\right)\right] & \text{if} \quad z < 0 \\ \alpha_z R_{s||} \quad \text{if} \quad z = 0 \\ 0 \quad \text{if} \quad z > 0 \end{cases}$$

$$\alpha_z = \frac{\omega_r}{2Q} \qquad \bar{\omega} = \sqrt{\omega_r^2 - \alpha_z^2}$$

Examples of wakes/impedances
Equations of resonators

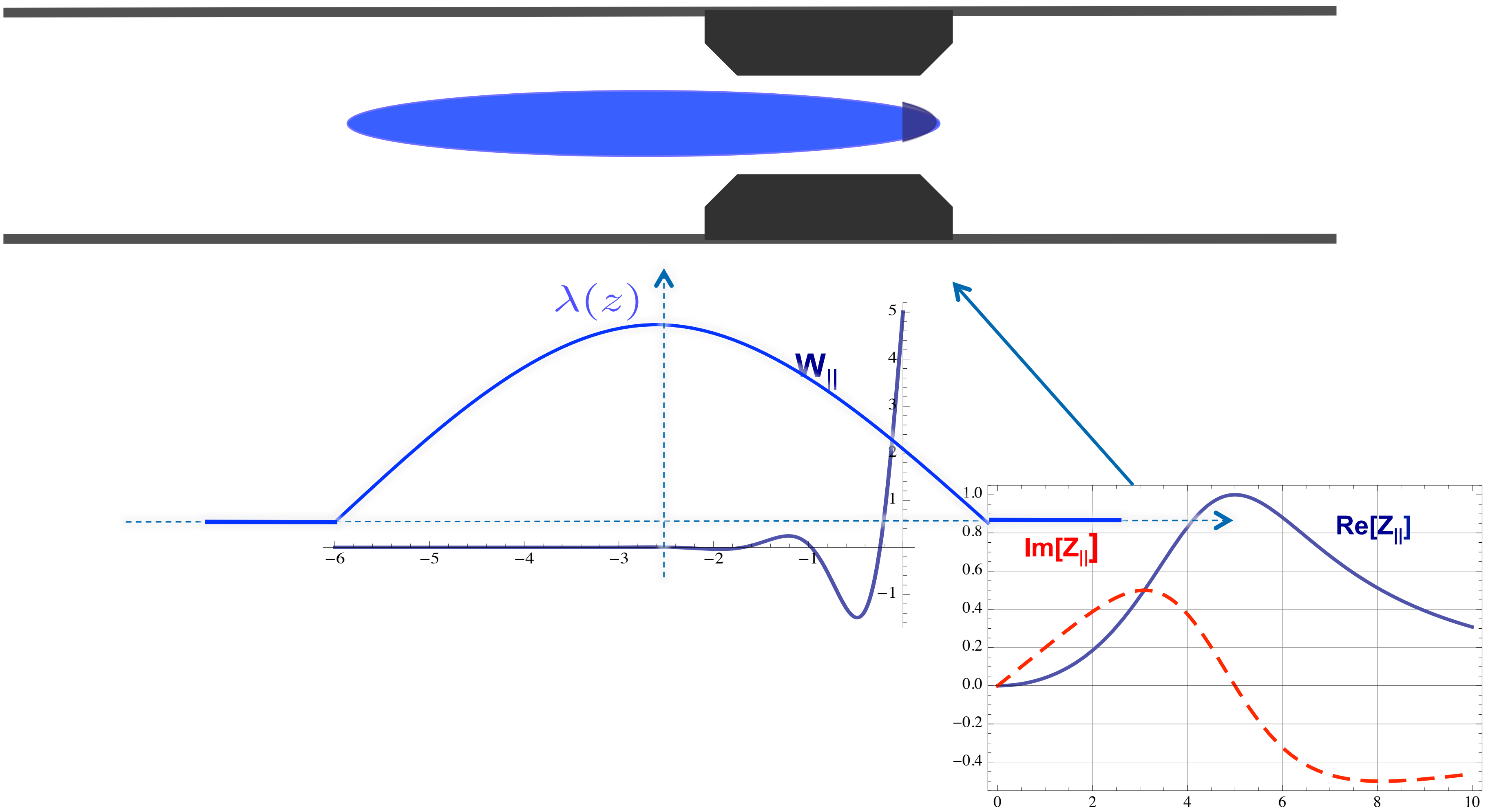
$$Z_{x,y}^{\text{Res}}(\omega) = \frac{\omega_r}{\omega} \frac{R_{s(x,y)}}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

$$W_{x,y}^{\text{Res}}(z) = \begin{cases} \frac{R_{s(x,y)}\omega_r^2}{Q\bar{\omega}} \exp\left(\frac{\alpha_t z}{c}\right) \sin\left(\frac{\bar{\omega} z}{c}\right) & \text{if} \quad z < 0 \\ 0 \quad \text{if} \quad z \geq 0 \end{cases}$$

$$\alpha_t = \frac{\omega_r}{2Q} \qquad \bar{\omega} = \sqrt{\omega_r^2 - \alpha_t^2}$$

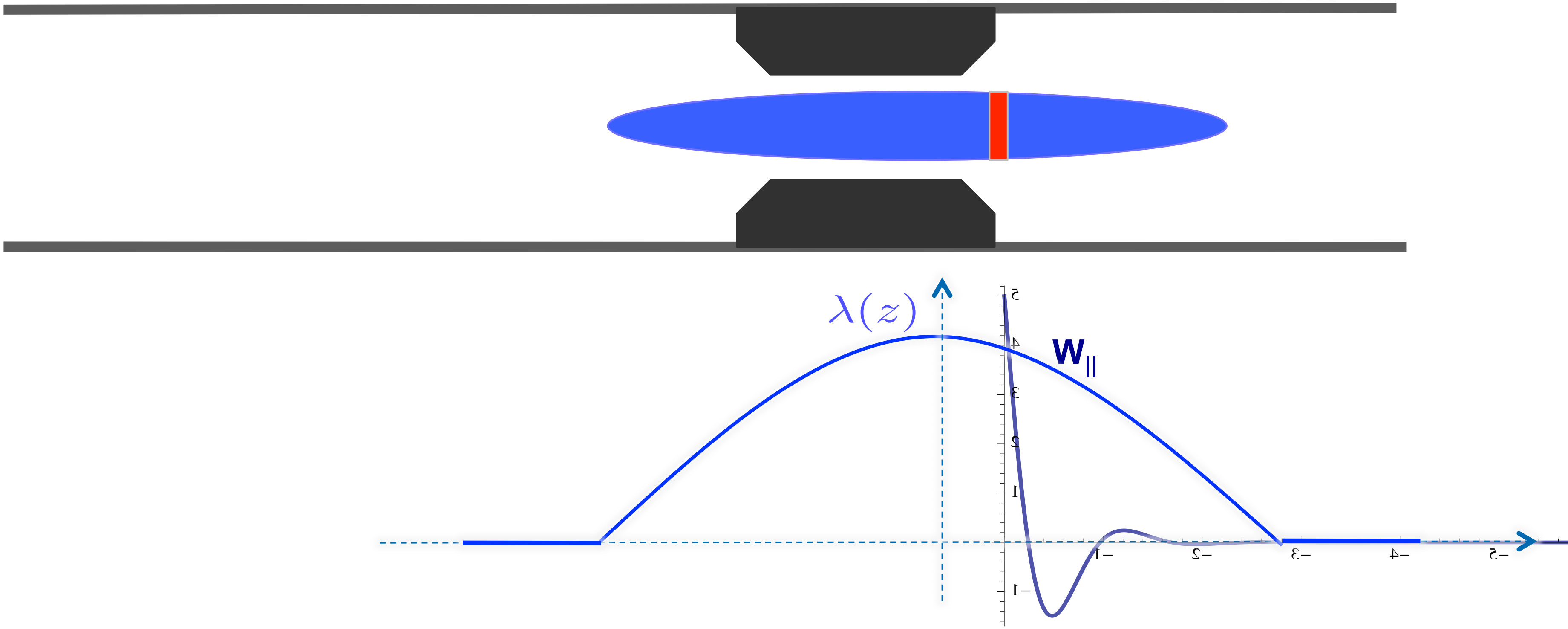
Single bunch vs. multi-bunch effects

Single bunch going through a
broad-band resonator



Single bunch vs. multi-bunch effects

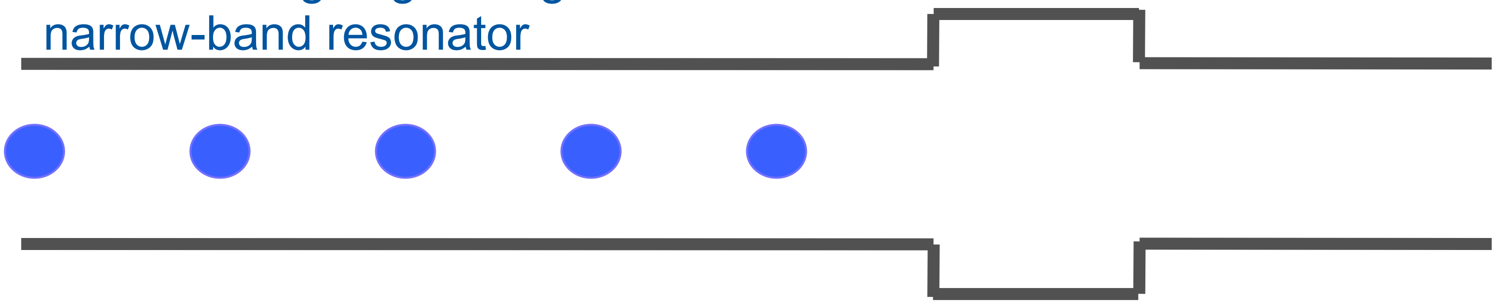
Single bunch going through a broad-band resonator

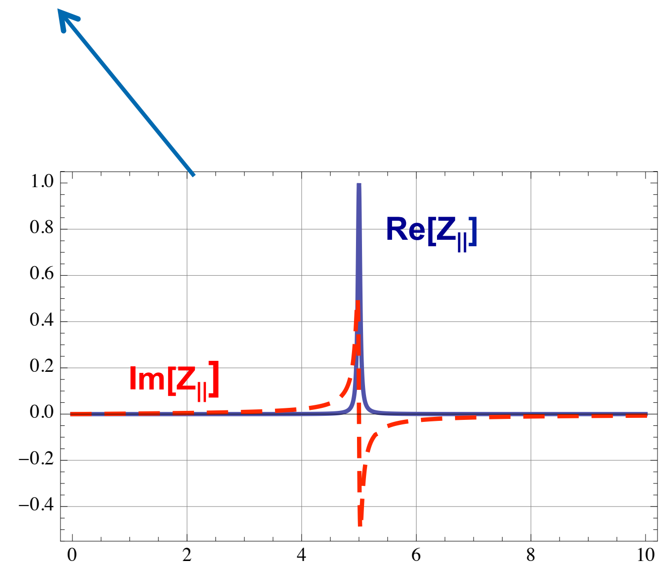


$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_{||}(z - z') dz'$$

Single bunch vs. multi-bunch effects

Bunch train going through a narrow-band resonator





Single bunch vs. multi-bunch effects

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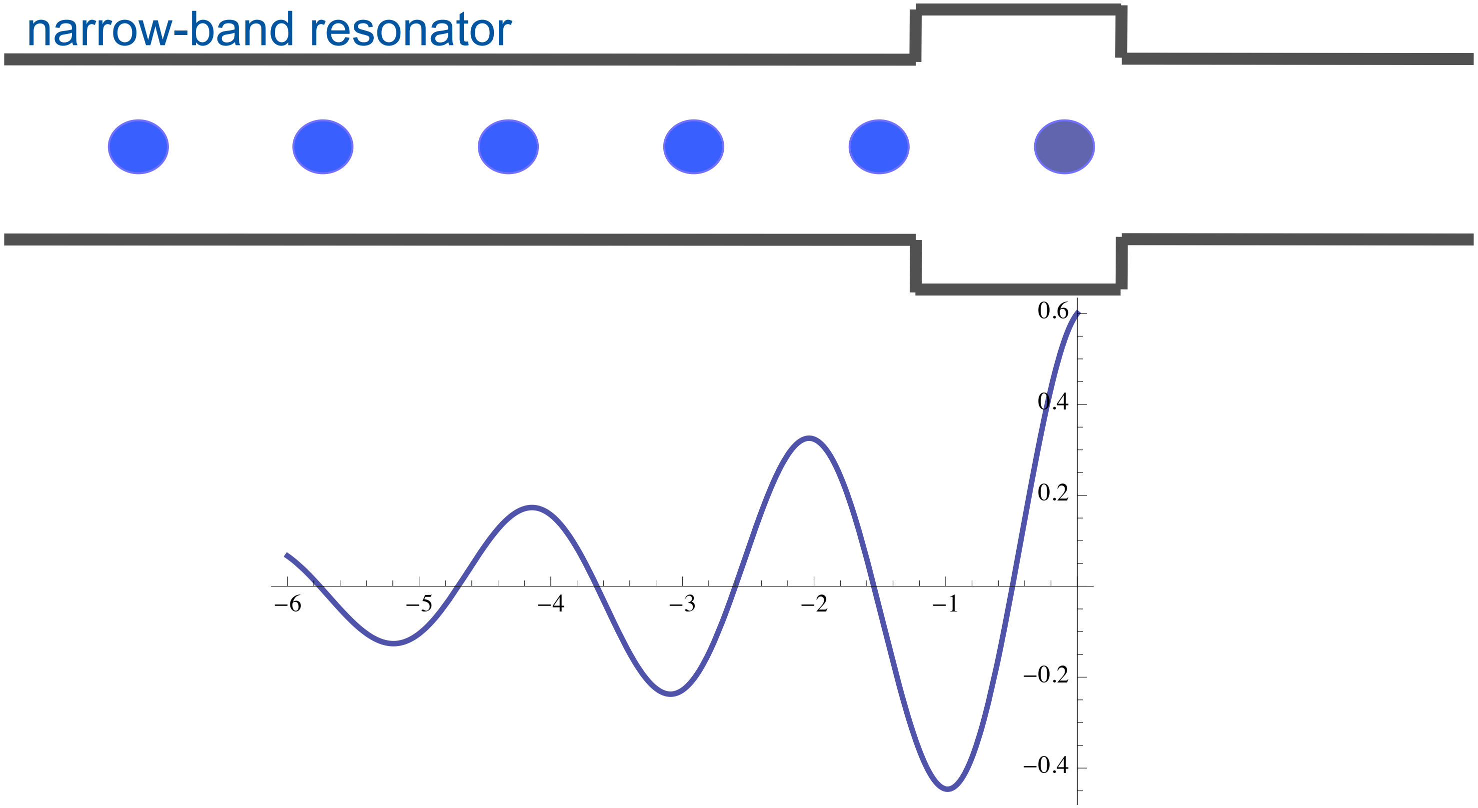
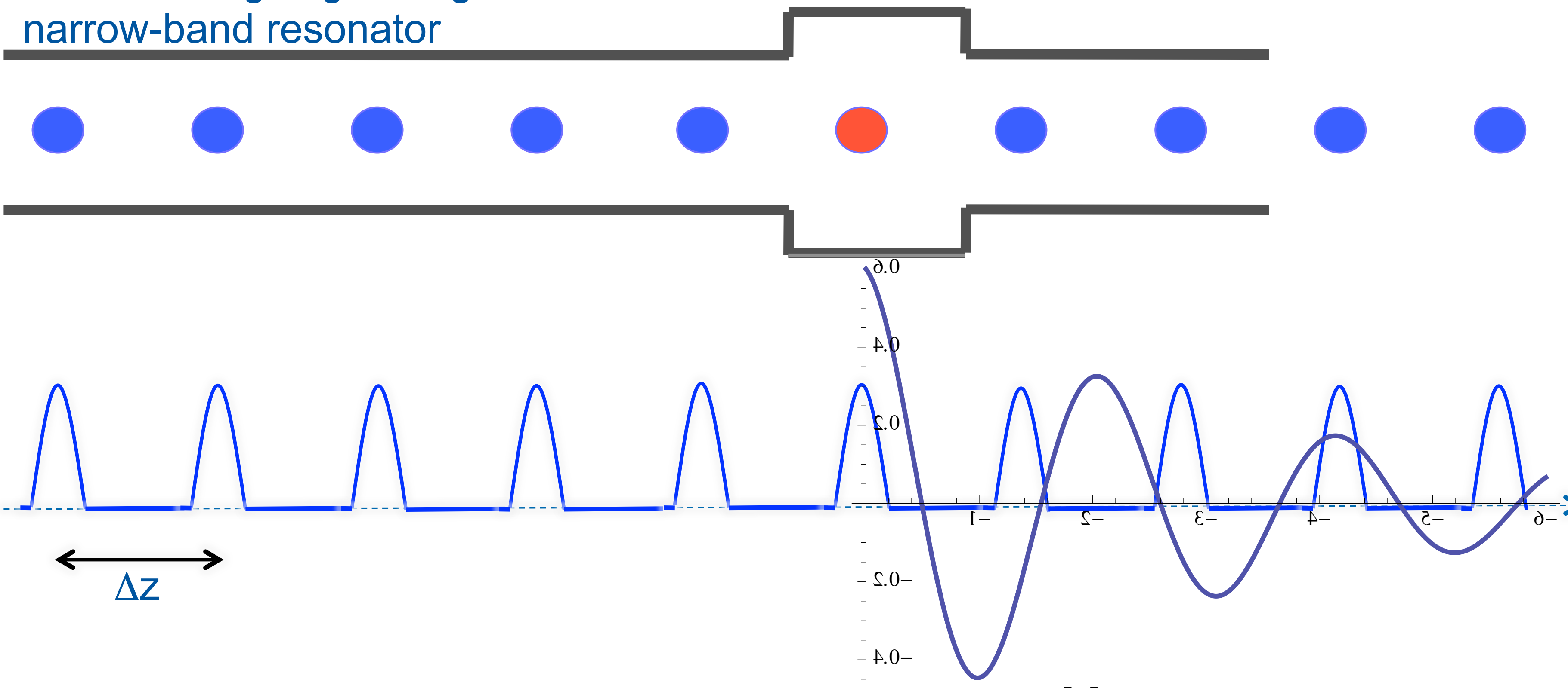


Single bunch vs. multi-bunch effects

Bunch train going through a narrow-band resonator

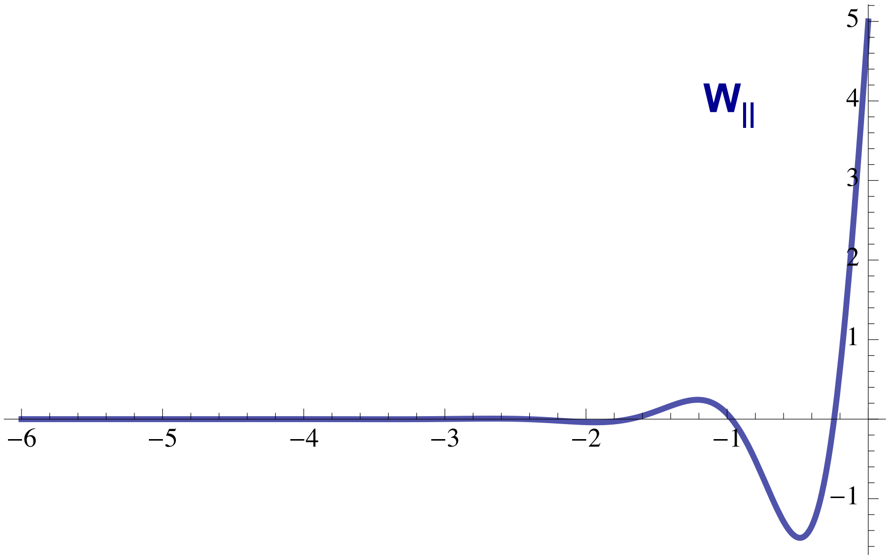


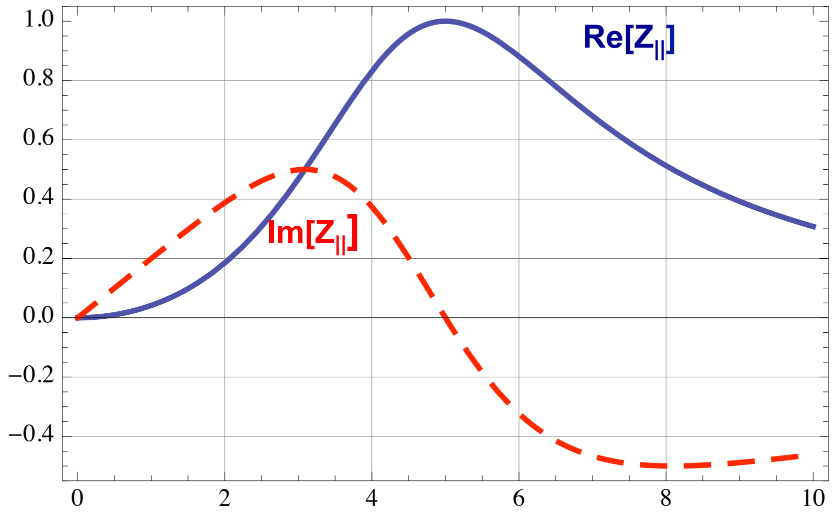
$$\Delta E_j = -e^2 \int_{z_j}^{\infty} \lambda(z')W_{||}(z_j - z')dz' \approx N_j e^2 \sum_{i=0}^{M} N_i W[(j-i)\Delta z]$$

$$\Delta E_j = -e^2 \int_{z_j}^{\infty} \lambda(z')W_{||}(z_j - z')dz' \approx N_j e^2 \sum_{k=0}^{\infty} \sum_{i=0}^{M} N_i W[(j-i)\Delta z + kC]$$

Single bunch vs. multi-bunch effects

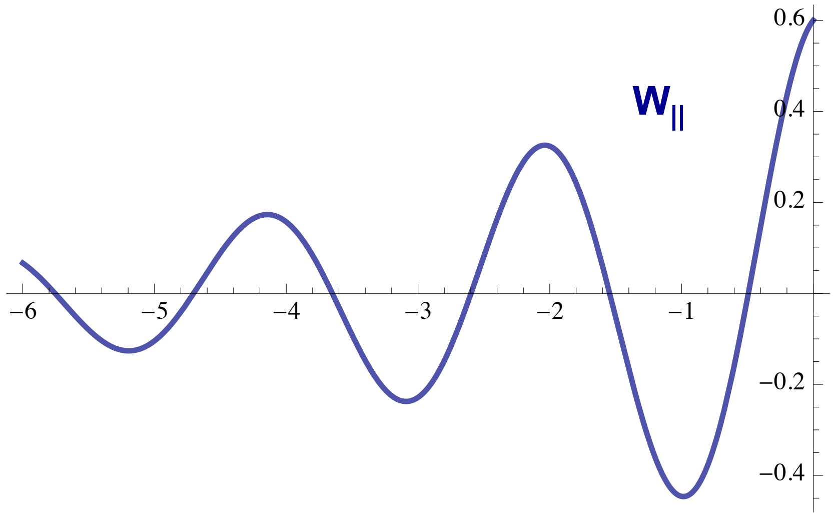
– A short-lived wake, decaying over the length of one bunch, can only cause intra-bunch (head-tail) coupling
– It can be therefore responsible for single bunch collective effects

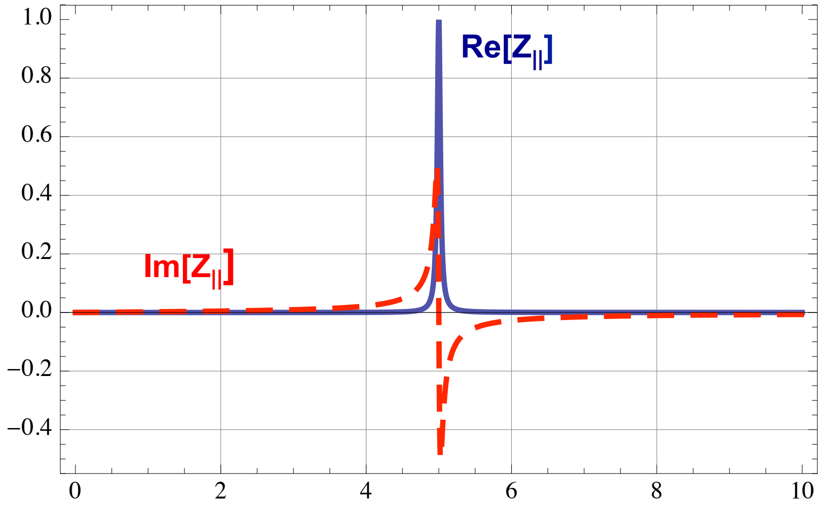




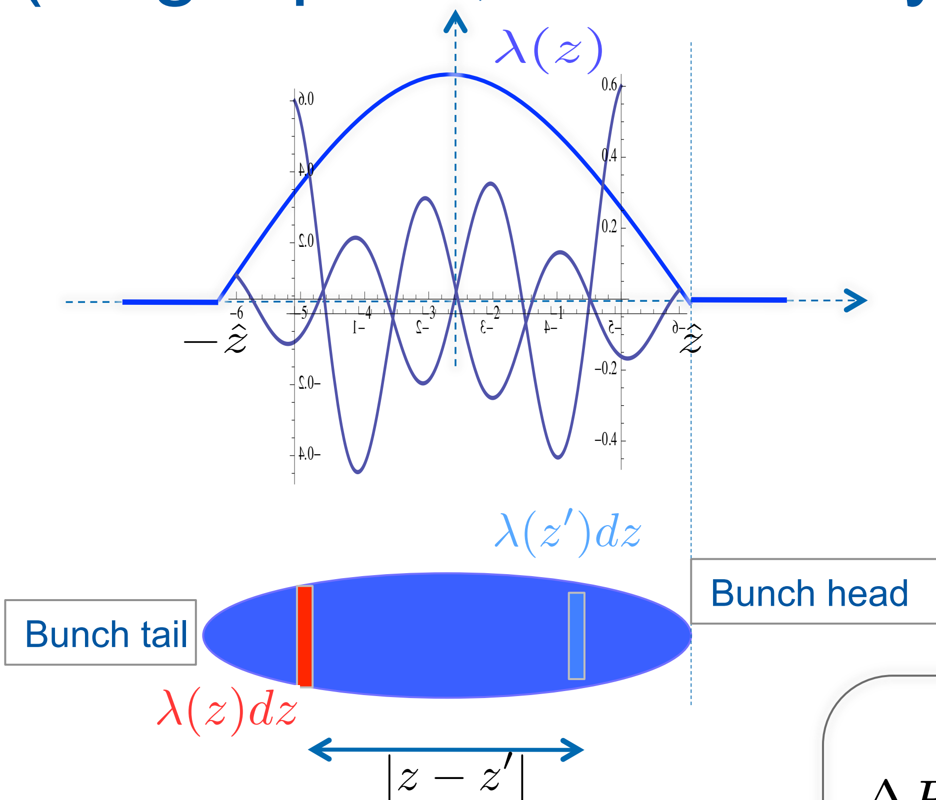
Single bunch vs. multi-bunch effects

- A long-lived wake field, decaying over the length of a train of bunches, or even several turns, causes bunch-to-bunch or multi-turn coupling
- It can be therefore responsible for multi-bunch or multi-turn collective effects





Energy loss of a bunch (single pass, no memory)



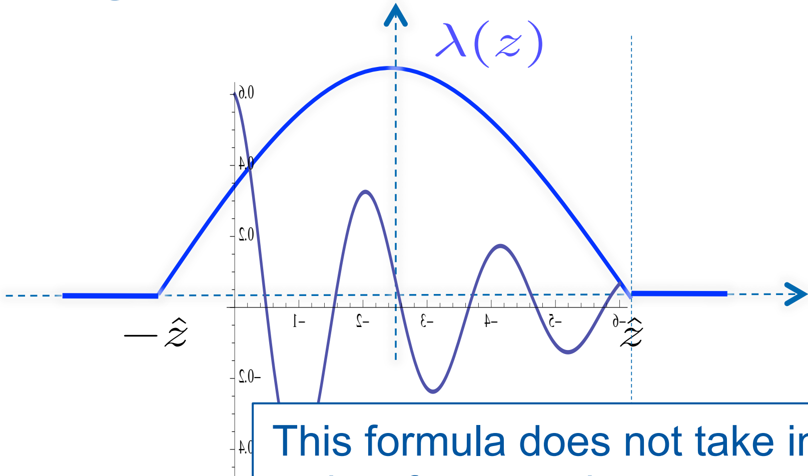
$$\Delta E(z, z') = -e^2 \lambda(z') W_{||}(z - z') dz'$$

$$\Delta E(z) = -e^2 \int_z^{\hat{z}} \lambda(z') W_{||}(z - z') dz'$$

$$\Delta E = \int_{-\hat{z}}^{\hat{z}} \lambda(z) \Delta E(z) dz$$

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \tilde{\lambda}(\omega) \tilde{\lambda}^*(\omega) Z_{||}(\omega) d\omega = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_{||}(\omega) \right] d\omega$$

Energy loss of a bunch (single pass, no memory)



$$\Delta E(z, z') = -e^2 \lambda(z') W_{||}(z - z') dz'$$

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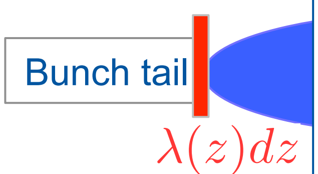

This formula does not take into account the contribution of the wakes from previous passages in case of periodic traversals.

Therefore, it holds if:

1. The bunch only goes once through the wake source
2. The wake is sufficiently short-lived that it has fully decayed at the time of the next traversal

$$\Delta E = -\frac{e^2}{2\pi} \int_{-\infty}^{\infty} \tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega) Z_{||}(\omega) d\omega =$$

$$-\frac{e^2}{2\pi} \int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_{||}(\omega)\right] d\omega$$

Energy loss of a bunch (single pass with memory)

$$\Delta E = -\frac{e^2}{2\pi}\int_{-\infty}^{\infty}\lambda(z)dz\int_{-\infty}^{\infty}dz'\lambda(z')\sum_{k=-\infty}^{\infty}W_{||}(kC+z-z')dz'$$

$$\sum_{k=-\infty}^{\infty}W_{||}(kC+z-z') = \frac{c}{C}\sum_{p=-\infty}^{\infty}Z_{||}(p\omega_0)\exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}Z_{||}(p\omega_0)\underbrace{\int_{-\infty}^{\infty}\lambda(z)\exp\left(\frac{-ip\omega_0 z}{c}\right)dz}_{\tilde{\lambda}(p\omega_0)}\underbrace{\int_{-\infty}^{\infty}\lambda(z')\exp\left(\frac{ip\omega_0 z'}{c}\right)dz'}_{\tilde{\lambda}^*(p\omega_0)}$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

Energy loss of a bunch (single pass with memory)

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$$\sum_{k=}^{\infty} W_{||}(kC+z-z') = \frac{c}{C}\sum^{\infty} Z_{||}(p\omega_0)\exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

$$\Delta E = -\frac{e^2\omega_0}{2\pi}$$

$$\left(\frac{ip\omega_0 z'}{c}\right)dz'$$

This formula is more general.
It still calculates the energy loss for single pass but taking into account the contribution of the wakes from previous passages in case of periodic traversals.

In the case of a circular machine, this formula expresses the bunch energy loss per turn

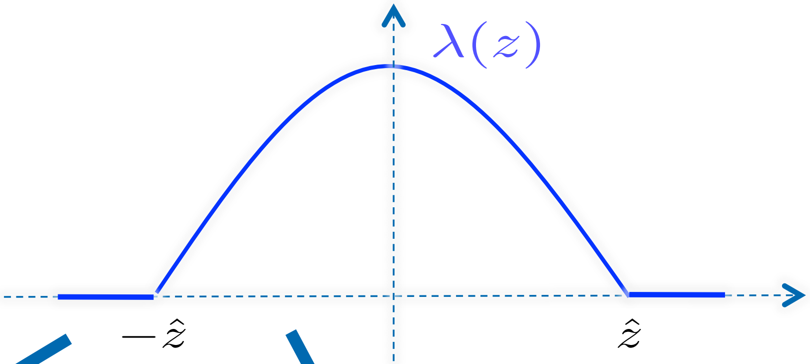
$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

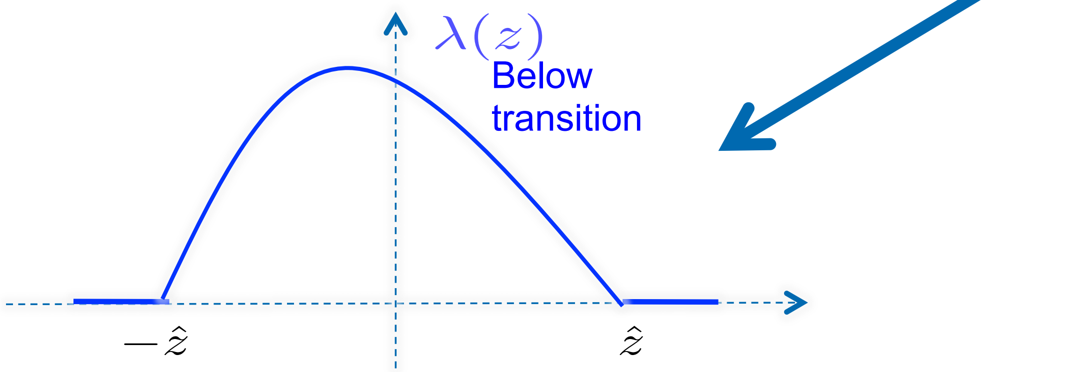
Bunch energy loss per turn and stable phase

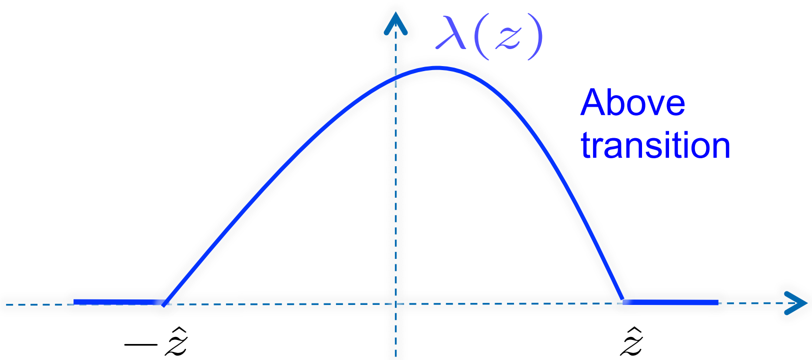
- The RF system compensates for the energy loss by imparting a net acceleration to the bunch
- Therefore, the bunch readjusts to a new equilibrium distribution in the bucket and moves to an average synchronous angle $\Delta\Phi_s$

$$\sin \Delta\Phi_s = \frac{\Delta E_{\rm turn}}{NeV_m} =$$

$$= \frac{e\omega_0}{2\pi N V_m} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 {\rm Re}\left[Z_{||}(p\omega_0)\right]$$





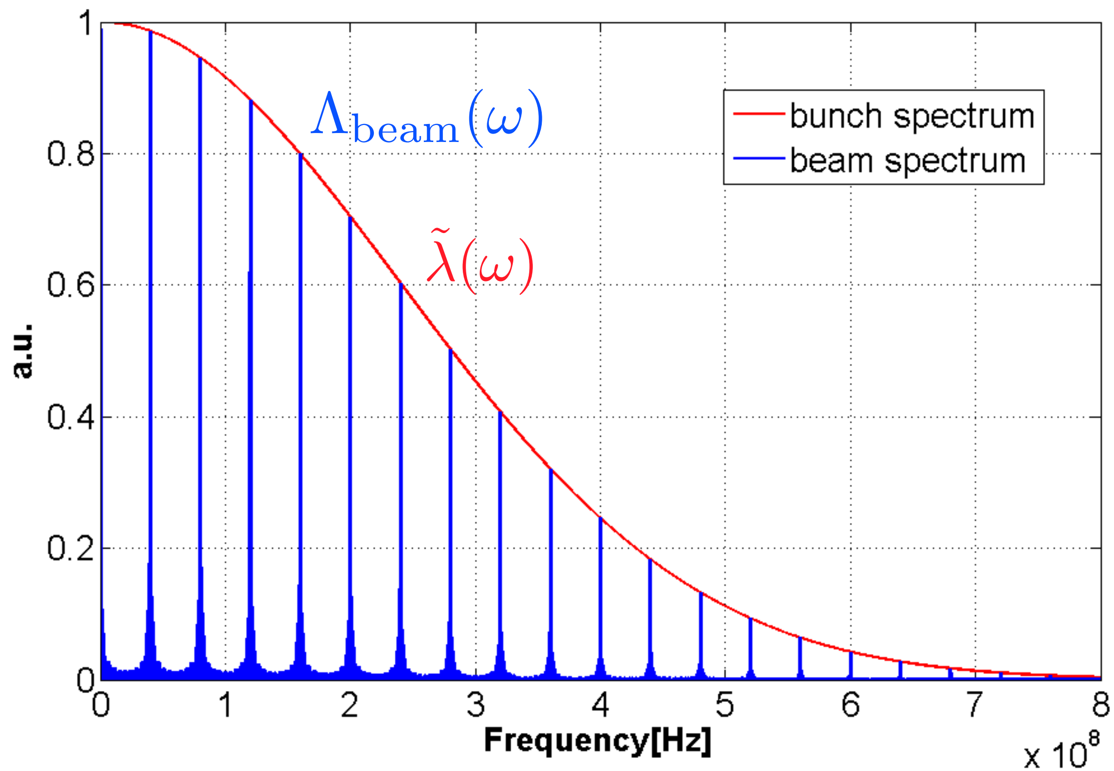


Beam energy loss per turn

$$\Delta E_{\text{turn}} - \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2\cdot \mathrm{Re}[Z_{||}(p\omega_0)]$$

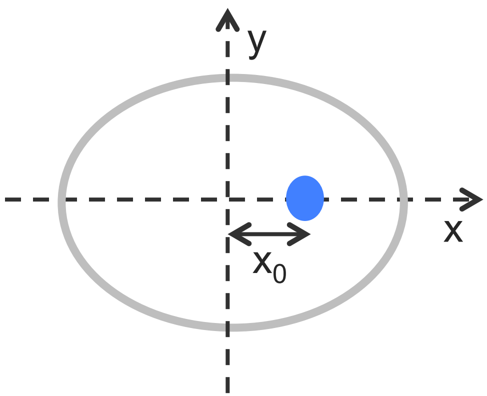
Bunch spectrum
$\tilde{\lambda}(\omega)$

Beam spectrum
$\Lambda_{\text{beam}}(\omega)$

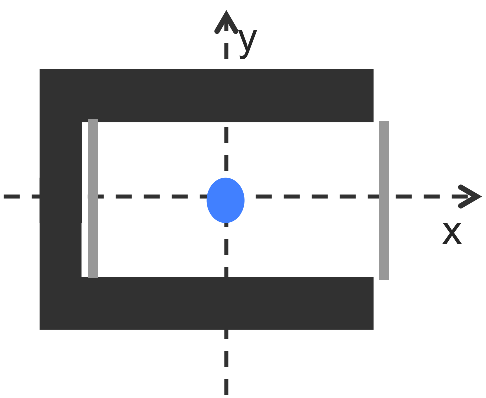


Beam deflection kick

Off-axis traversal of symmetric chamber



Traversal of asymmetric chamber



$$\Delta x'(z) = -\frac{e^2 x_0}{E_0} \int_{-\hat{z}}^{\hat{z}} \lambda(z') \left[W_x(z-z') + W_{Qx}(z-z')\right] dz'$$

$$\Downarrow$$

$$\langle \Delta x' \rangle = -\frac{e^2 x_0}{\pi E_0} \int_0^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Im}[Z_x(\omega) + Z_{Qx}(\omega)] d\omega$$

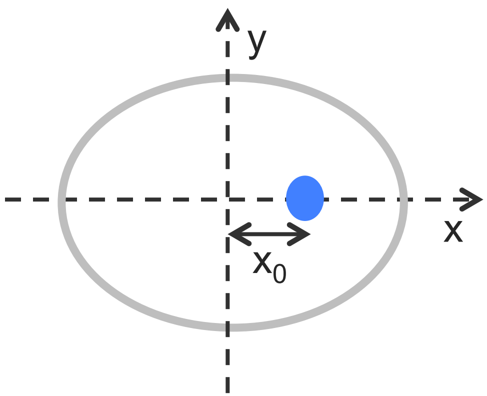
or

$$\langle \Delta x' \rangle = -\frac{e^2 x_0 \omega_0}{2\pi E_0} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \mathrm{Im}[Z_x(p\omega_0) + Z_{Qx}(p\omega_0)]$$

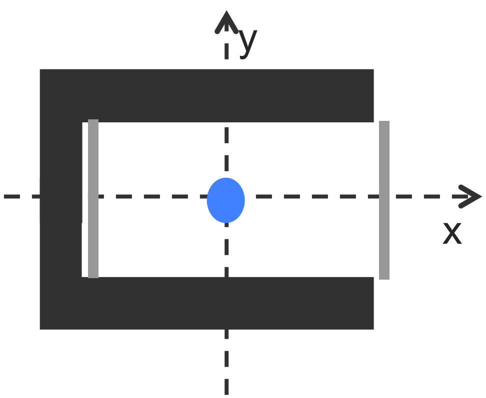
$$\langle \Delta x' \rangle = -\frac{e^2 \omega_0}{2\pi E_0} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \mathrm{Im}[Z_{Cx}(p\omega_0)]$$

Beam deflection kick

Off-axis traversal of symmetric chamber



Traversal of asymmetric chamber



$$\Delta x'(z) = -\frac{e^2 x_0}{E_0} \int_{-\hat{z}}^{\hat{z}} \lambda(z') \left[W_x(z-z') + W_{Qx}(z-z') \right] dz'$$

$$\Downarrow$$

$$\langle \Delta x' \rangle = -\frac{e^2 x_0}{\ldots} \int^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Im}[Z_x(\omega) + Z_{Qx}(\omega)] d\omega$$

The beam deflection kicks
- ⇒ Are responsible for intensity dependent orbit variations
- ⇒ Cause z-dependent orbits and can determine tilted equilibrium bunch distributions for long bunches

$$\ldots) + Z_{Qx}(p\omega_0)]$$

$$\langle \Delta x' \rangle = -\frac{e^2 \omega_0}{2\pi E_0} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \mathrm{Im}[Z_{Cx}(p\omega_0)]$$

Some hints for energy loss estimations

$$\lambda(z) = \frac{N}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \tilde{\lambda}(\omega) = N \exp\left(-\frac{\omega^2\sigma_z^2}{2c^2}\right)$$

$$\int_{-\infty}^{\infty} |\tilde{\lambda}(\omega)|^2 \mathrm{Re}\left[Z_{||}(\omega)\right] d\omega$$

can be calculated

1) With $Z_{||}(\omega) = Z_{||}^{Res}(\omega)$ from slide 77 in the two limiting cases

$$\sigma_z \gg \frac{c}{\omega_r}$$

Need to expand $\mathrm{Re}[Z_{||}(\omega)]$ for small ω

$$\sigma_z \ll \frac{c}{\omega_r}$$

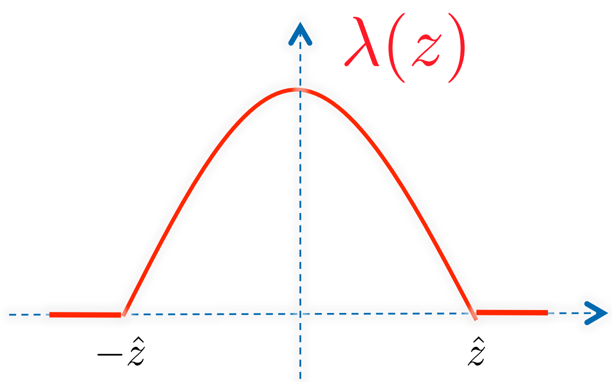
Need to assume $|\lambda(\omega)|$ constant over $\mathrm{Re}[Z_{||}(\omega)]$

2) With $Z_{||}(\omega) = Z_{||RW}(\omega)$ from slide 64

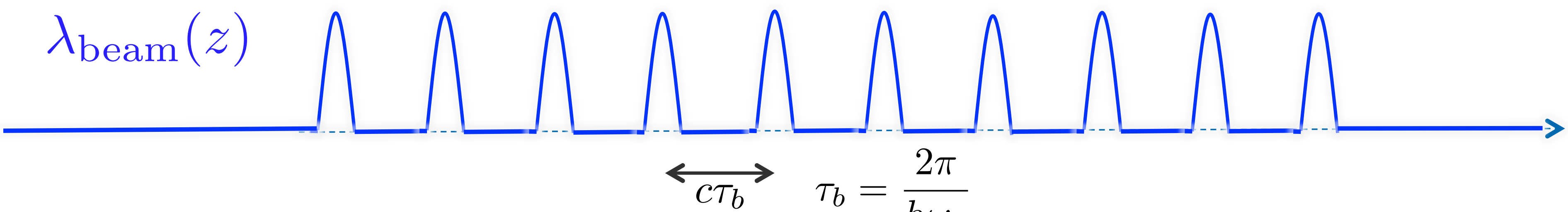
Some applications of the energy loss

- Train of bunches
  - Analytical derivation
  - Example 1: Heating of the SPS extraction kickers
  - Example 2: Heating of the LHC beam screen
- Beams with exotic bunch spacings
  - The case of "doublets"
  - Example: Impact in LHC, e.g. on the TDI

Energy loss of a train of bunches



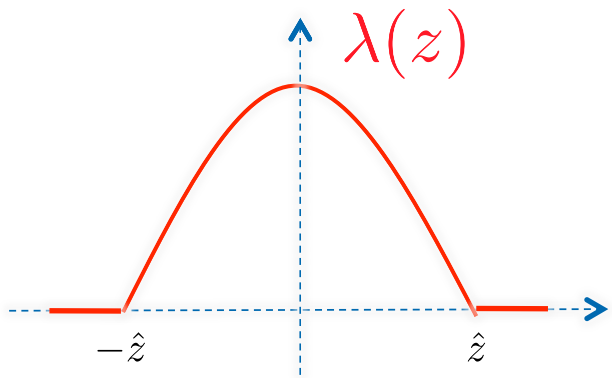
A train of *M* identical equally spaced bunches circulating in a ring



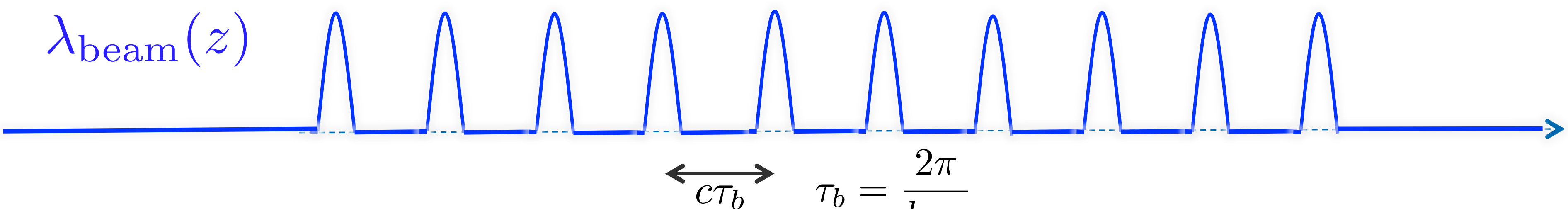
$$\tau_b = \frac{2\pi}{h\omega_0}$$

$$\lambda_{\rm beam}(z) = \sum_{n=0}^{M-1} \lambda(z - nc\tau_b) \quad \overset{\mathcal{F}}{\Longleftrightarrow} \quad \Lambda_{\rm beam}(\omega) = \tilde{\lambda}(\omega) \sum_{n=0}^{M-1} \exp(-in\omega\tau_b)$$

Energy loss of a train of bunches



A train of *M* identical equally spaced bunches circulating in a ring



$$\tau_b = \frac{2\pi}{h\omega_0}$$

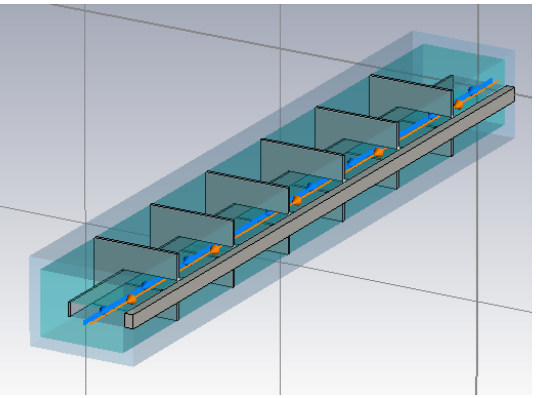
$$\Delta E_{\text{beam}} = \frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \text{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos(\frac{2\pi M p}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

Energy loss of a train of bunches

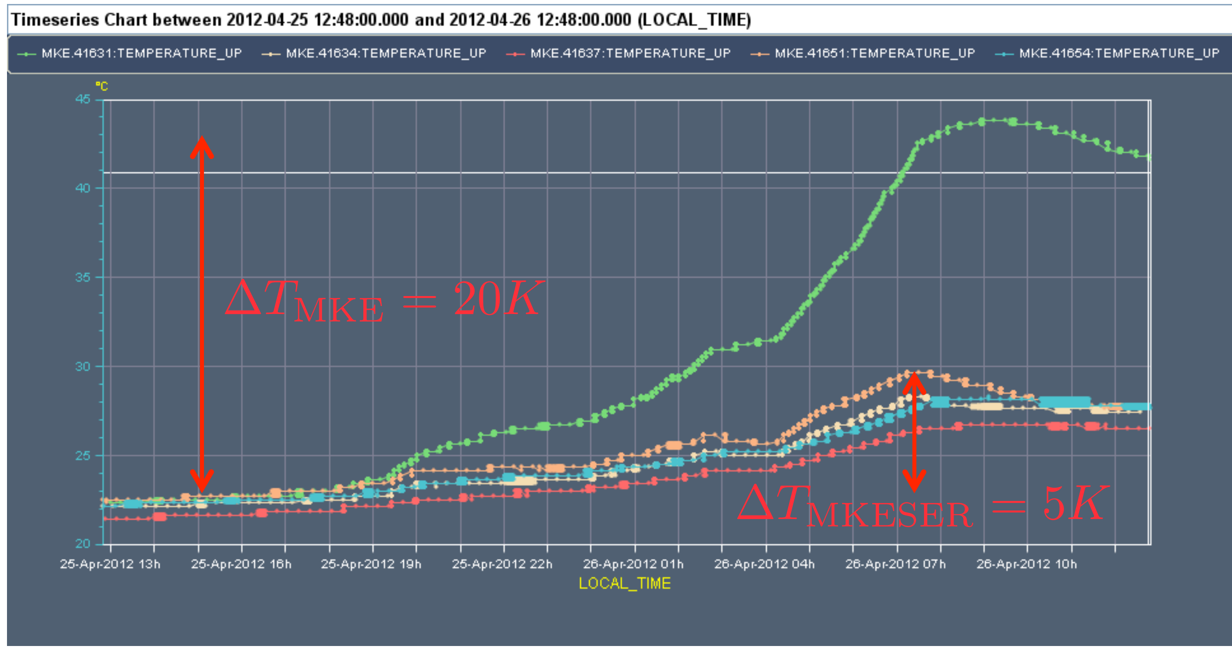
$$\Delta E_{\text{beam}} = \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2 \text{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos(\frac{2\pi Mp}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

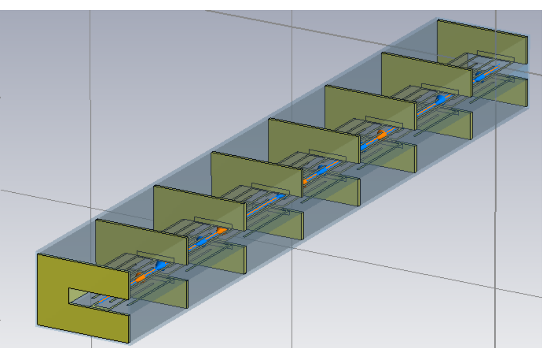
- The potential leading terms in the summation are those with $p = k \cdot h$, as the ratio in brackets tends to M^2.
- Narrow-band impedances peaked around multiples of the harmonic number of the accelerator are the most efficient to drain energy from the beam ➔ beam induced heating, instabilities.
- This type of impedances, usually associated to the RF systems and their higher order modes (HOMs), need mitigation in the accelerator design (e.g. detuners, HOM absorbers).

Application to the SPS extraction kickers



Running multi-bunch (25 ns) for several hours causes significant heating of these kickers

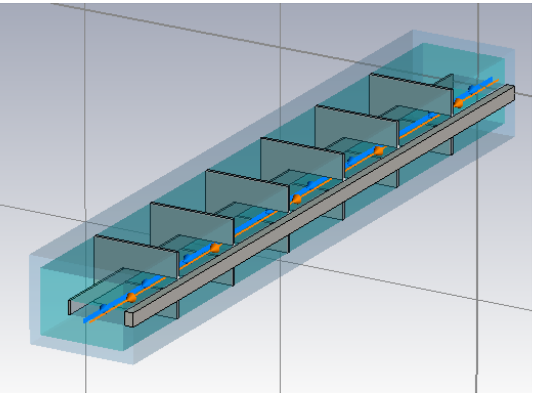


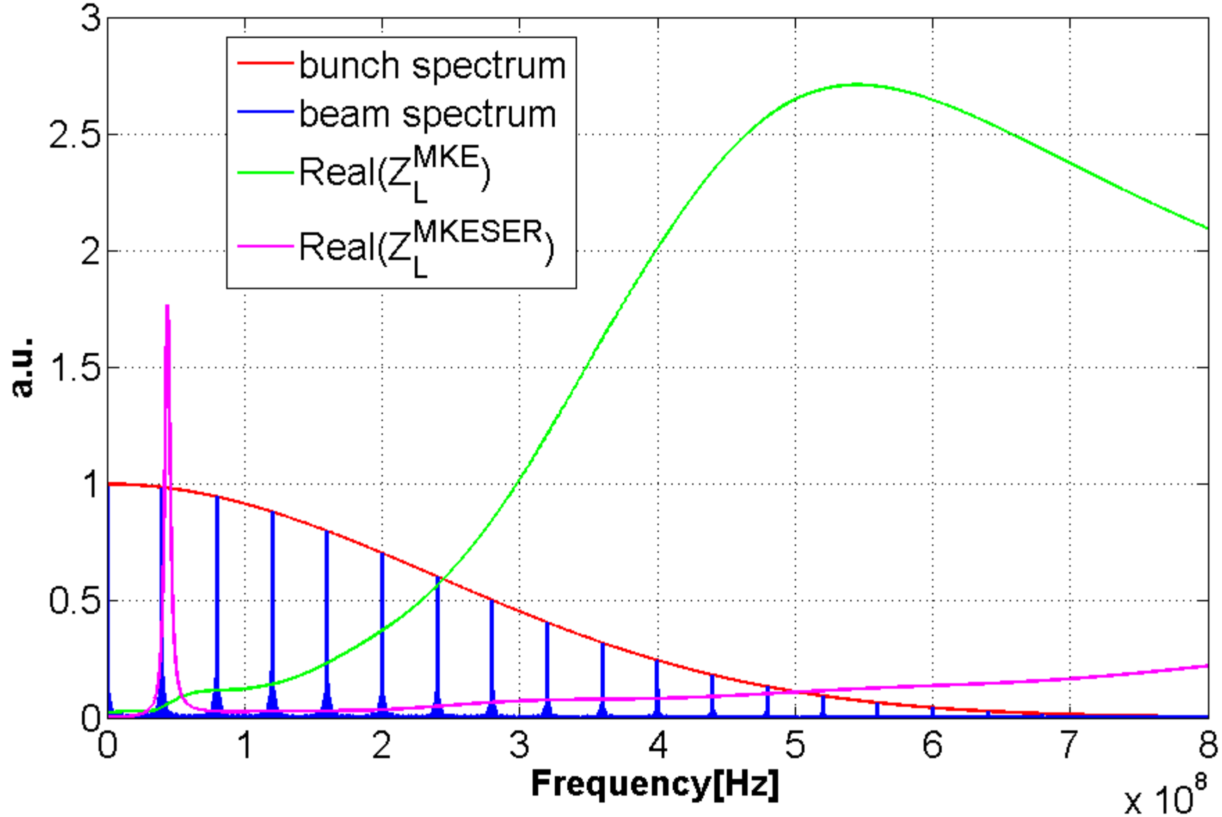


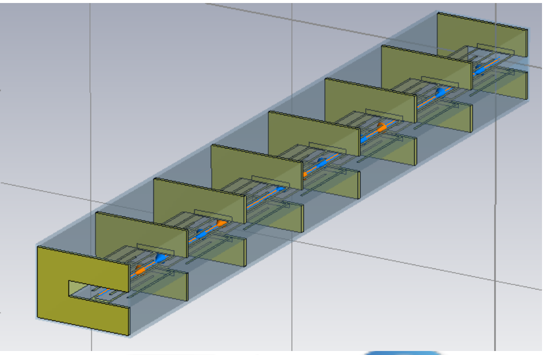
~ 17h run with 25 ns beams
at 26 GeV after technical stop

$$\frac{\Delta T_{\rm MKE}}{\Delta T_{\rm MKESER}} = \frac{\Delta W_{\rm MKE}}{\Delta W_{\rm MKESER}} \approx 4$$

Application to the SPS extraction kickers





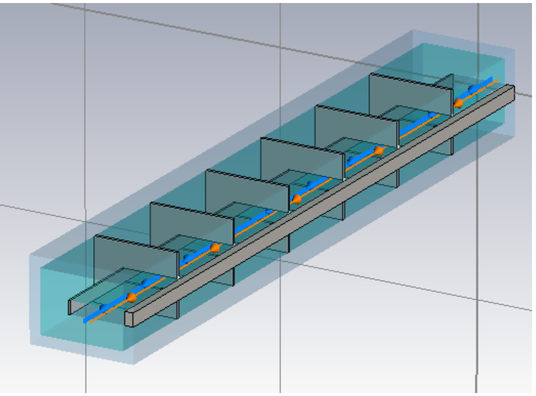


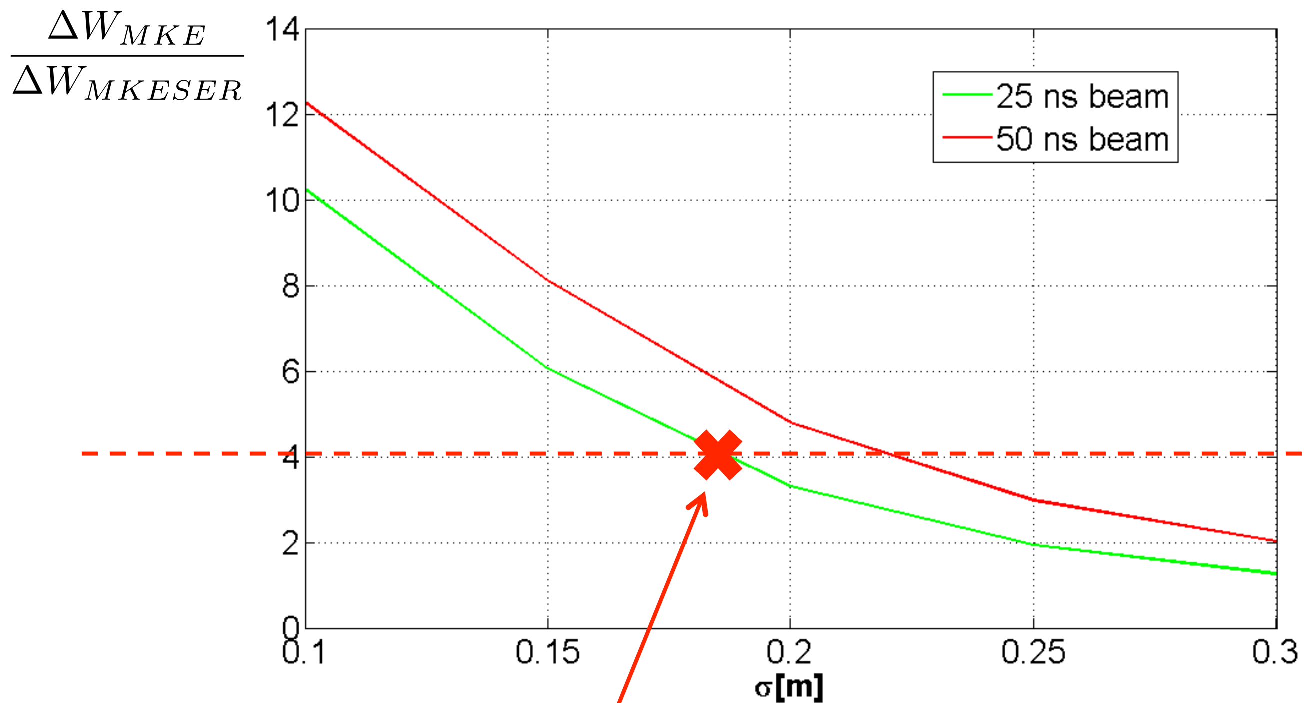
$$\Delta E_{\mathrm{beam}} = \frac{e^2\omega_0}{2\pi} \sum_{p=-\infty}^{\infty} |\tilde{\lambda}(p\omega_0)|^2 \mathrm{Re}\left[Z_{||}(p\omega_0)\right] \cdot \left[\frac{1-\cos(\frac{2\pi M p}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

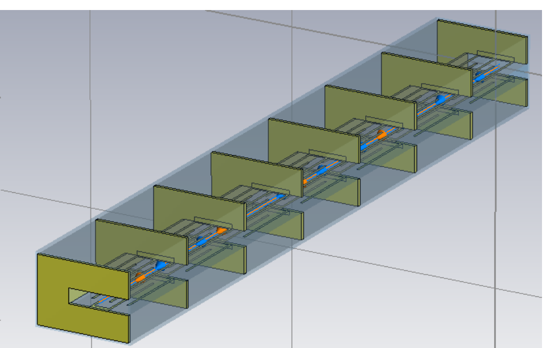
$$\Delta W = \frac{\Delta E_{\mathrm{beam}}}{T_0}$$

Z_L^{MKE}, Z_L^{MKESER}

Application to the SPS extraction kickers

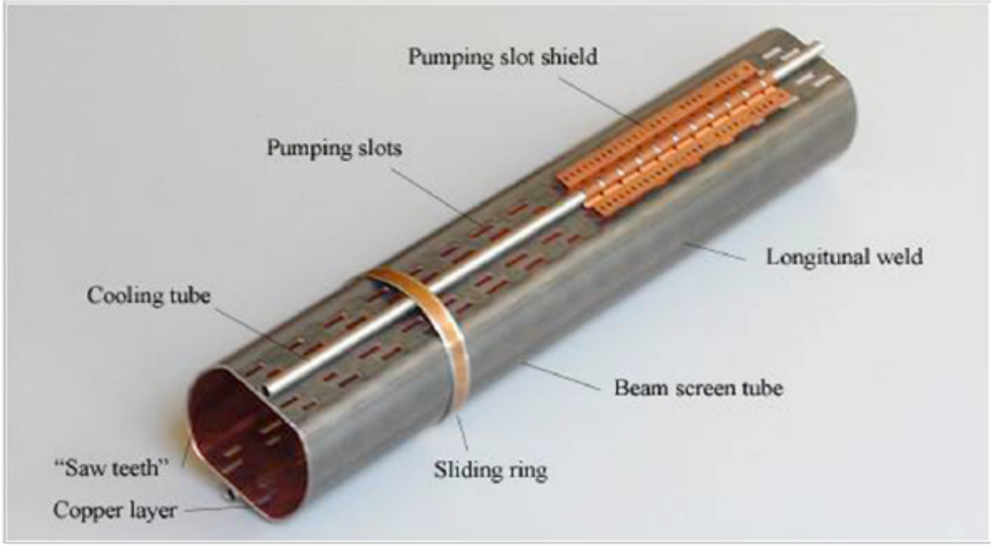


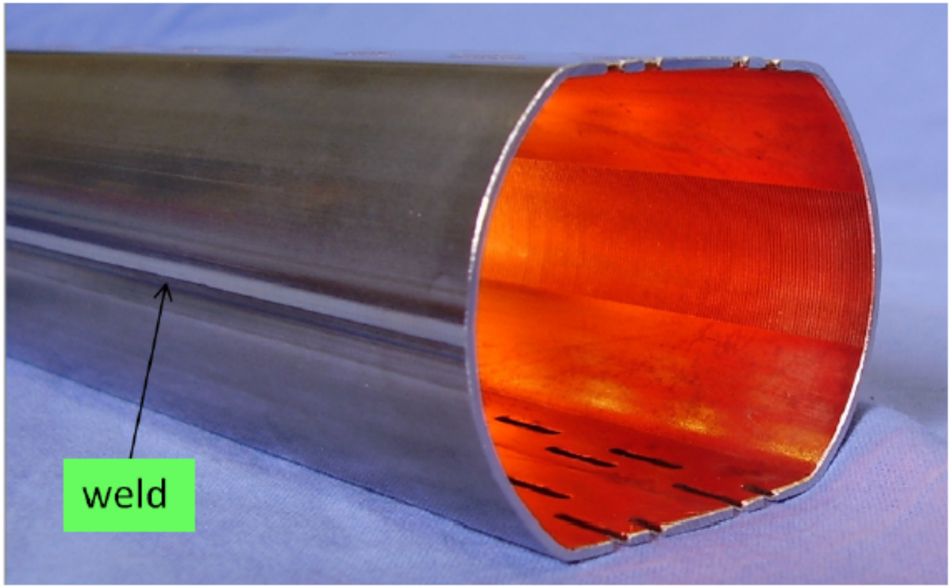




$$\frac{\Delta T_{\mathrm{MKE}}}{\Delta T_{\mathrm{MKESER}}} \approx 4$$

Application to the LHC beam screen





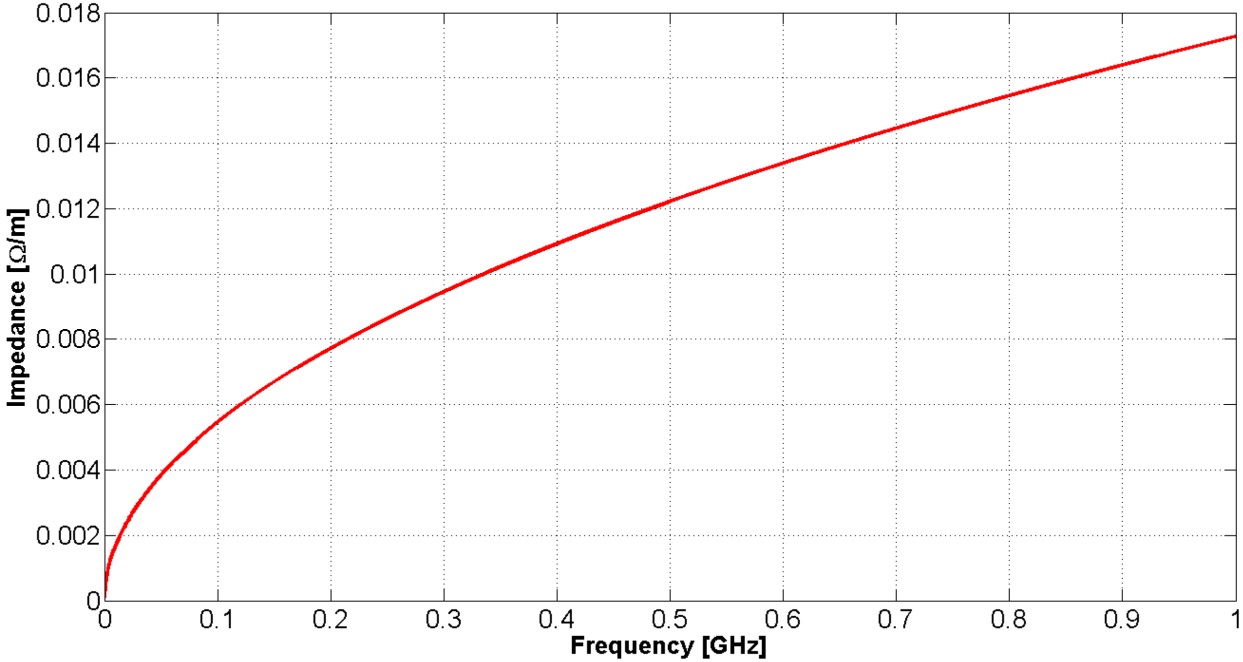
→ All along the arcs and in other cold regions of the LHC, a beam screen is interposed between the beam and the cold bore

→ The LHC beam screen is made of stainless steel with a layer of few μm of colaminated copper

→ Due to the production procedure, there is a stainless steel weld on one side of the beam screen that remains exposed to the beam.

→ The screen has holes for pumping on top and bottom

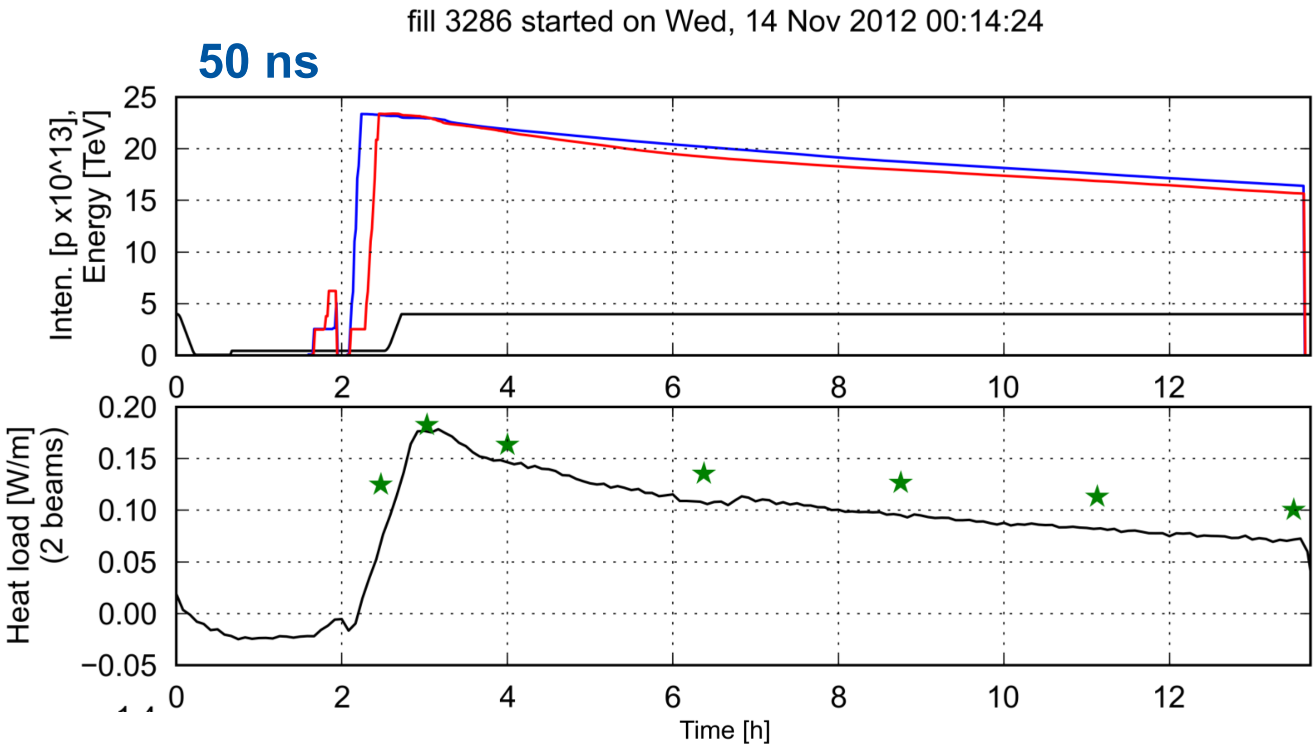
Application to the LHC beam screen



→ The impedance model includes the weld on one side of the beam screen, which means a small longitudinal stripe of exposed StSt, as well as the pumping holes

Application to the LHC beam screen

The heat dissipated on the beam screen can be calculated for a beam made of bunches spaced by 50 ns and compared to the measurement from cryogenics

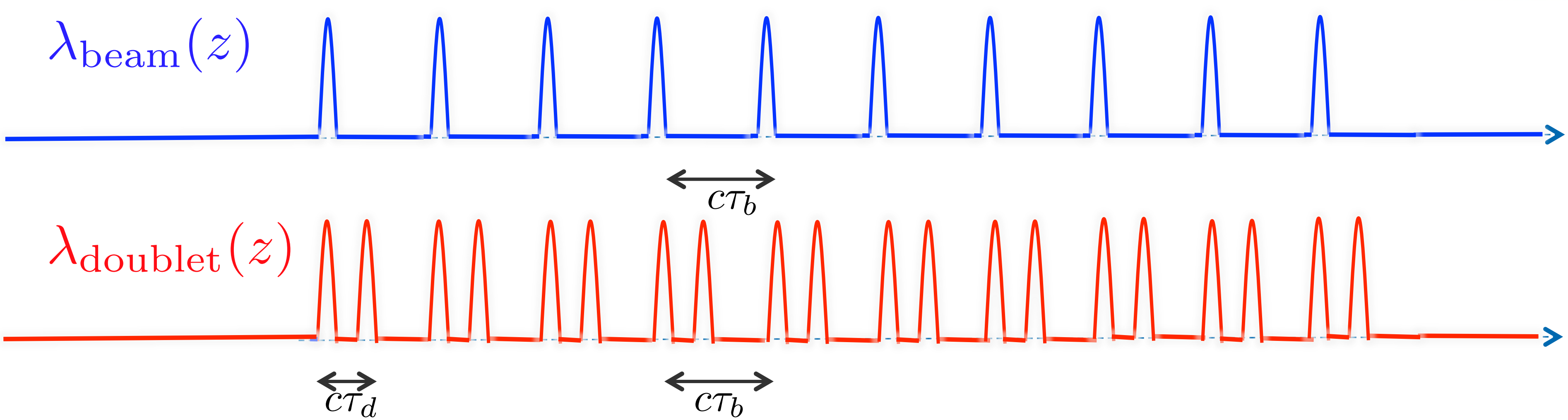


Heat load measurement from cryogenics

★ Estimation (impedance + synchrotron rad.)

$$\Delta E_{\text{beam}} = \frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}|\tilde{\lambda}(p\omega_0)|^2 \text{Re}\left[Z_{||}(p\omega_0)\right]\cdot\left[\frac{1-\cos(\frac{2\pi Mp}{h})}{1-\cos(\frac{2\pi p}{h})}\right]$$

Exotic bunch spacing: a "doublet" beam



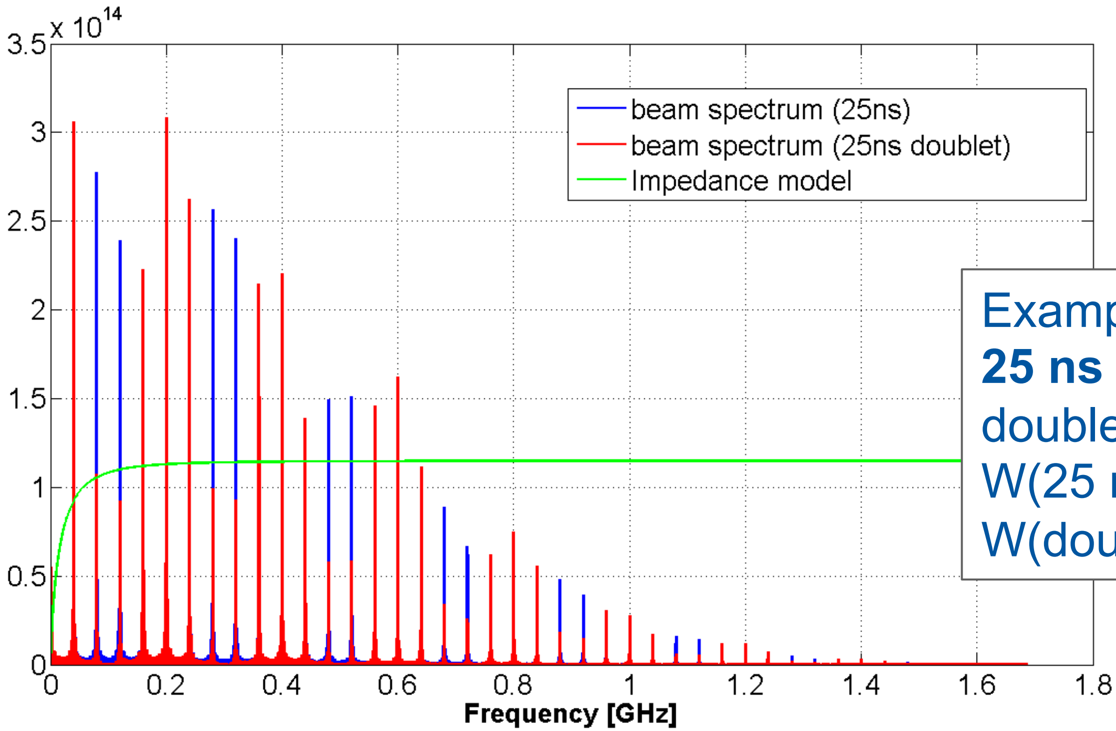
$$\Lambda_{\text{beam}}(\omega) \to \Lambda_{\text{doublet}}(\omega) = \Lambda_{\text{beam}}(\omega)\left[1 + \exp(-i\omega\tau_d)\right]$$

$$\Delta E_{\text{doublet}} = \frac{2e^2\omega_0}{\pi}\sum_{p=-\infty}^{\infty} |\Lambda_{\text{beam}}(p\omega_0)|^2 \cos^2\left(\frac{p\omega_0\tau_d}{2}\right) \text{Re}\left[Z_{||}(p\omega_0)\right]$$

N.B. in this example the doublet has double total intensity than single beam

Exotic bunch spacing: a “doublet” beam

- No additional impedance energy loss is expected with the doublet beam with respect to nominal beam for same total intensity
  - Beam power spectrum is modulated with $\cos^2$ function and lines are weakened by this modulation
  - For higher doublet intensity, global effect depends on the impedance spectrum
  - Example → LHC injection beam stopper (TDI)



Example for LHC beam in TDI:
25 ns (1.2 x 10^{11} p/b) vs. **20+5 ns** doublet (1.5 x 10^{11} p/doublet)
W(25 ns) = 456 W
W(doublet) = 338 W

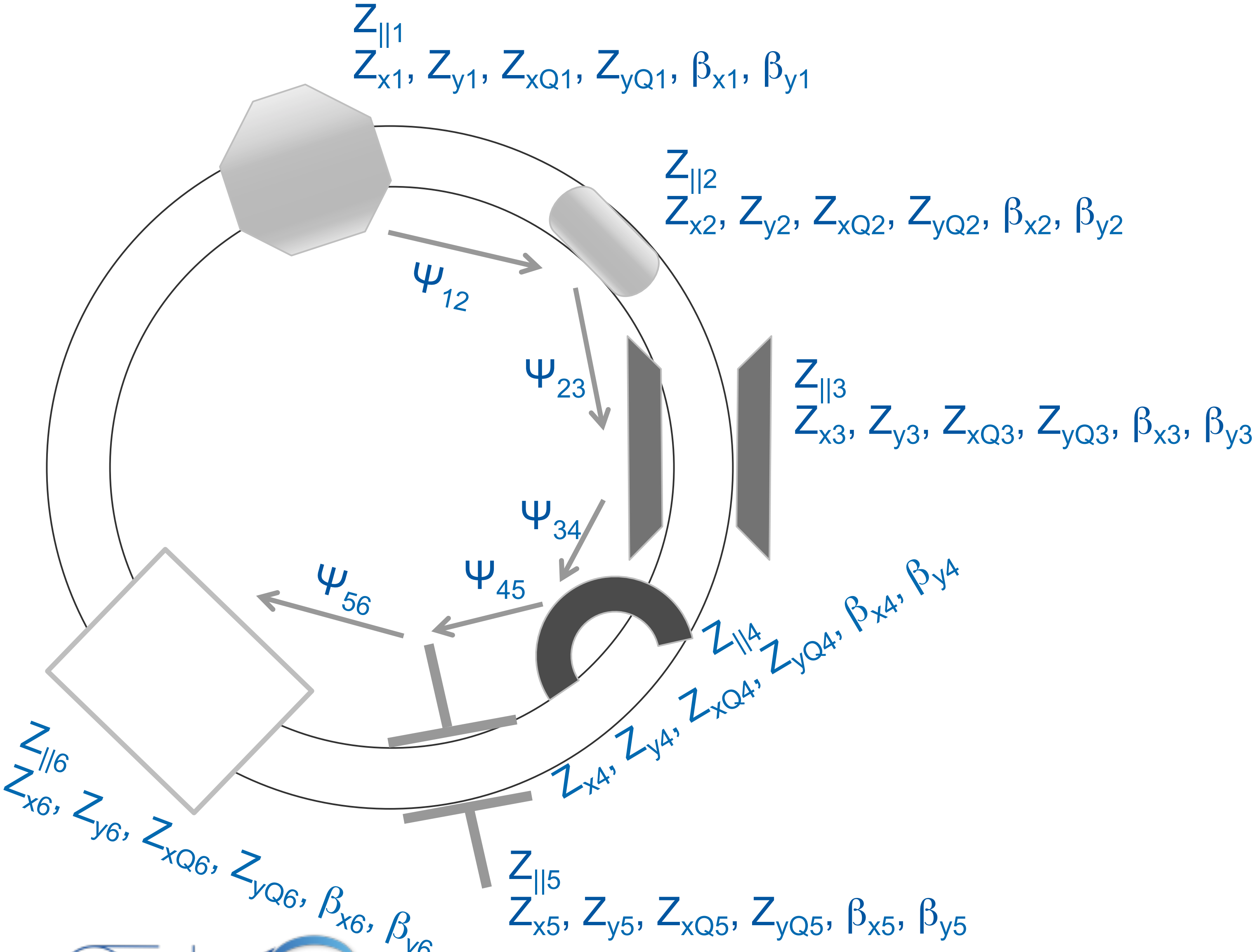
Impedance model of a machine

- To build the **impedance model of an accelerator** it is necessary to evaluate the impedance (wake) of all the elements (analytically, numerically, through measurements)
- The impedance model must then be bridged to beam dynamics studies
- When studying the beam dynamics of the machine, two approaches are possible:
  1. Each wake is separately applied to the beam particles and the beam is transported from one element to the next one with the correct phase advance (tricky when the effect is distributed, like for the resistive wall of the vacuum chamber)
  2. Assuming they are a small perturbation to the beam dynamics, all the wakes are summed up and the interaction of the beam with the impedance/wake is then lumped at one location ('kick approximation')
     - Longitudinal: The wake sum is directly used to change the momentum of all the beam particles
     - Transverse (both H and V): Each wake needs to first weighted by the beta function at the location of the wake source (like localized quadrupole errore) and then the weighted sum is divided by an average beta function. The resulting wake is then applied at a location with the average beta function

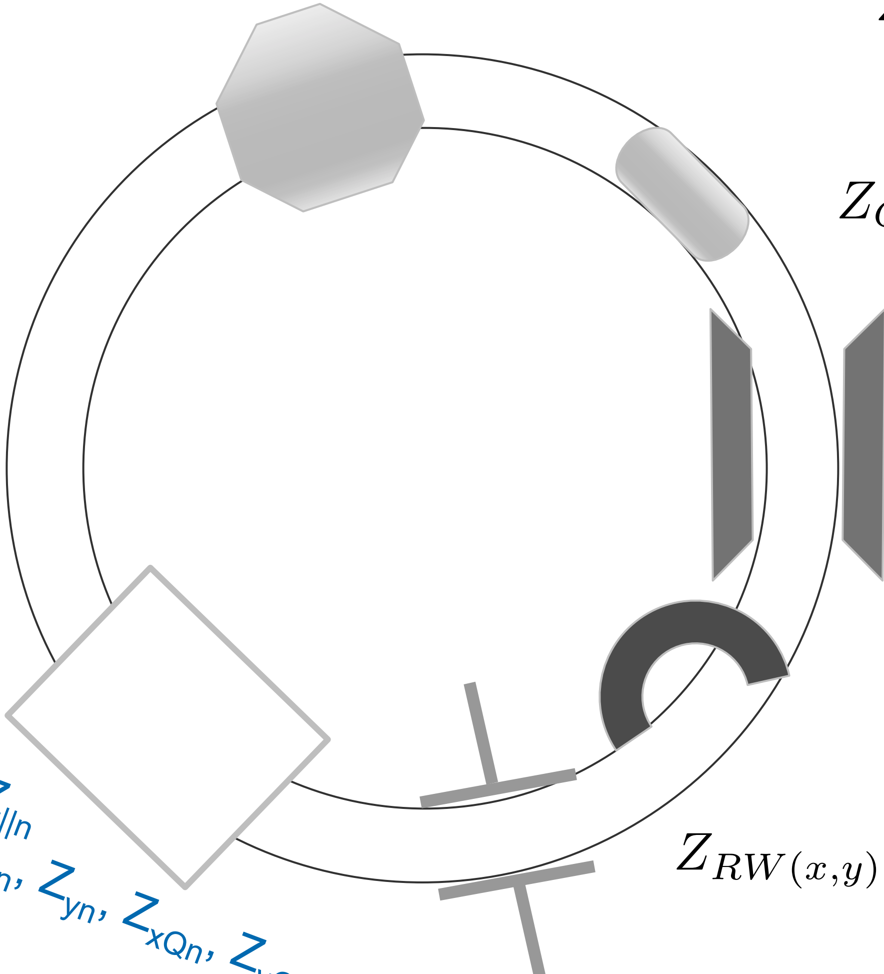
Impedance model of a machine

- The impedance model of the machine will contain therefore
  - A database of impedances of the individual accelerator elements (direct source of information for approach 1. and needed for localized heating studies)
  - A global wake/impedance table providing the (weighted) sum of the database elements to be used for beam dynamics following approach 2.
- In general, approach 1. is in general CPU-time-wise unviable (and anyway an overkill in most cases) and approach 2. is applied
- The impedance model of an accelerator is important
  - At the early stage of a machine life cycle, to monitor (and steer) that the global impedance of the machine under design/ construction is kept below the budget (i.e. maximum possible impedance to operate nominal beams in stable conditions)
  - In operation, to study limitations and target intensity upgrades (which usually entail a program of impedance mitigation/ reduction)

Impedance model of a machine



Impedance model of a machine



$$Z_{x,y}(\omega) = \frac{1}{\langle \beta_{x,y} \rangle} \sum_{n=1}^{M} \beta_{(x,y)n} Z_{(x,y)n}(\omega)$$

$$Z_{Qx,y}(\omega) = \frac{1}{\langle \beta_{x,y} \rangle} \sum_{n=1}^{M} \beta_{(x,y)n} Z_{Q(x,y)n}(\omega)$$

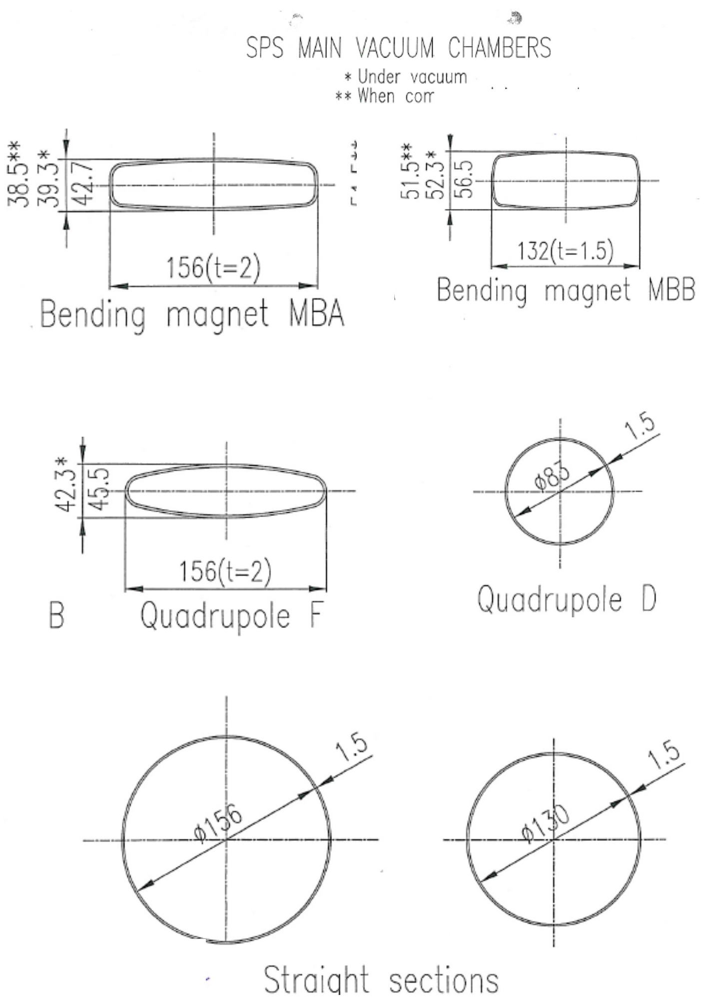
$$Z_{||}(\omega) = \sum_{n=1}^{M} Z_{||n}(\omega)$$

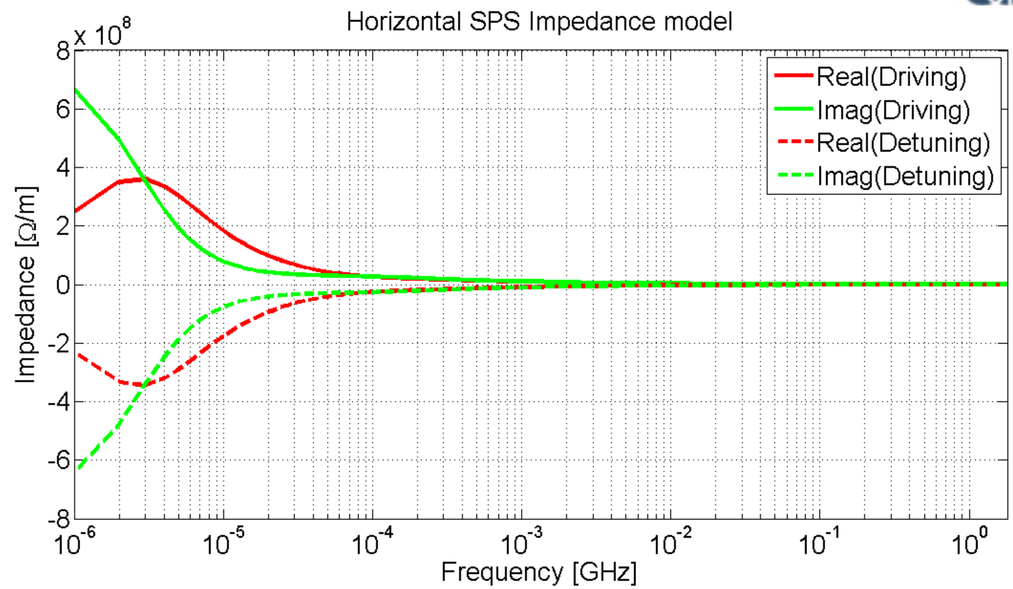
$$Z_{RW||}(\omega) = \oint Z_{RW||}(s;\omega) ds$$

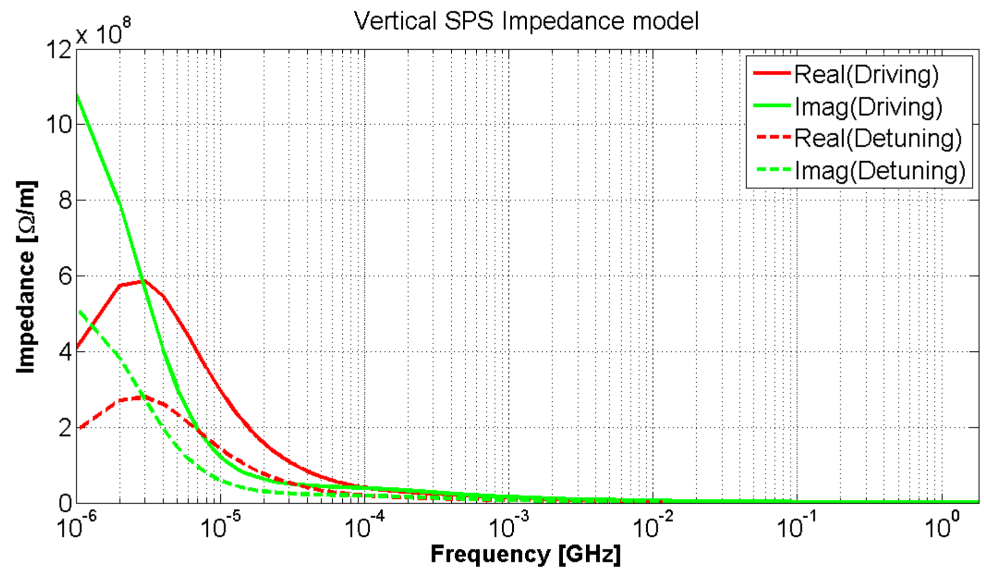
$$Z_{RW(x,y)}(\omega) = \frac{1}{\langle \beta_{x,y} \rangle} \oint \beta_{(x,y)}(s) Z_{RW(x,y)}(s;\omega) ds$$

$$Z_{RWQ(x,y)}(\omega) = \frac{1}{\langle \beta_{x,y} \rangle} \oint \beta_{(x,y)}(s) Z_{RWQ(x,y)}(s;\omega) ds$$

Impedance model of the SPS
The resistive wall

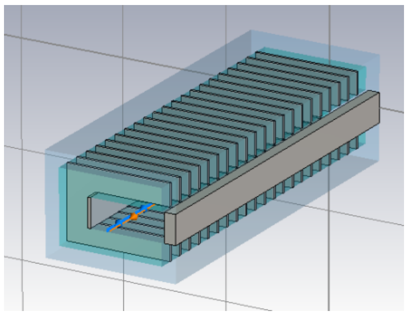


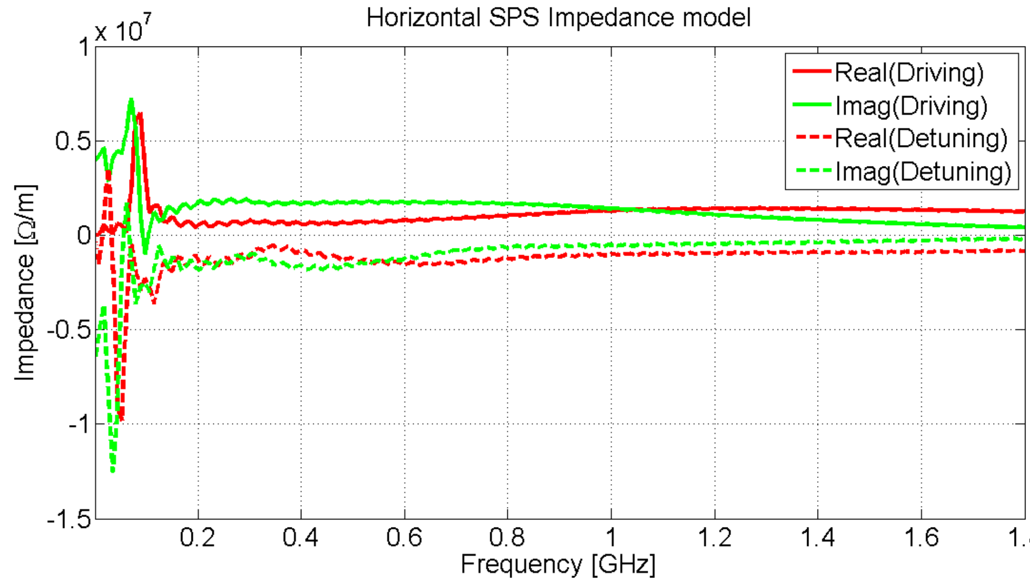




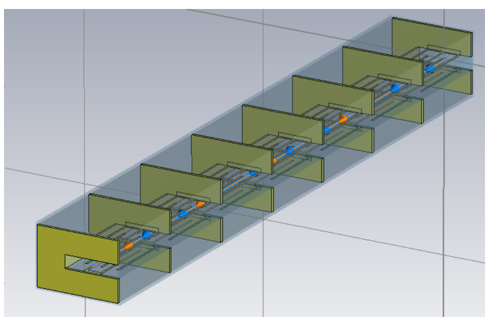
Impedance model of the SPS
The kickers

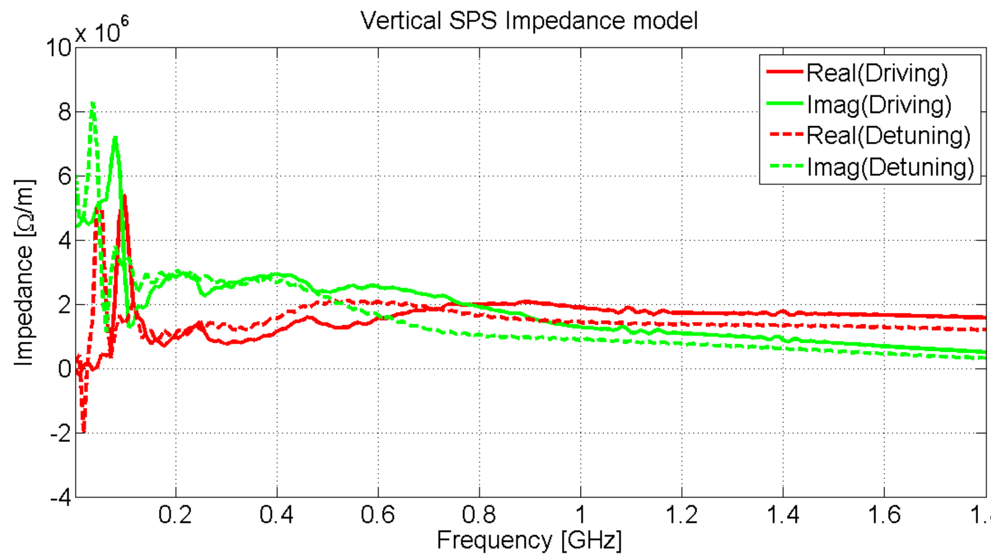
MKP





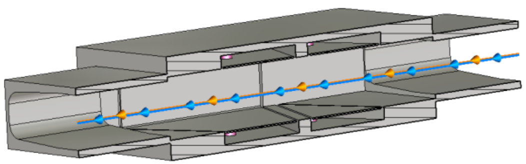
MKE



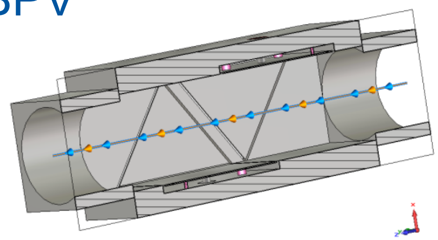


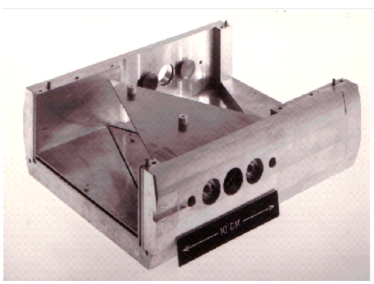
Impedance model of the SPS
The Beam Position Monitors

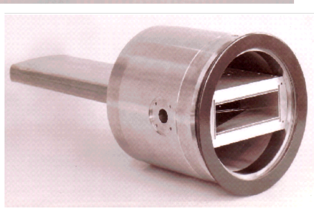
BPH

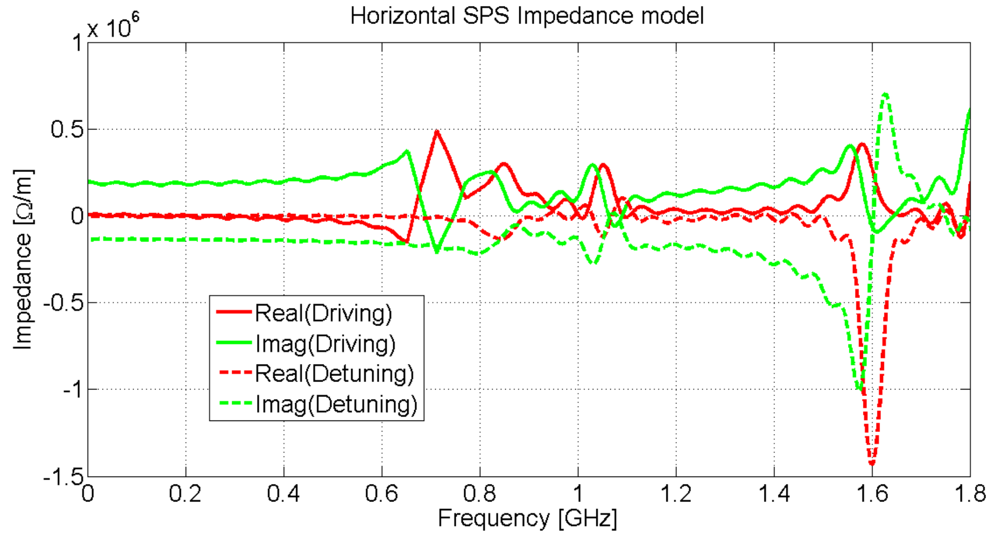


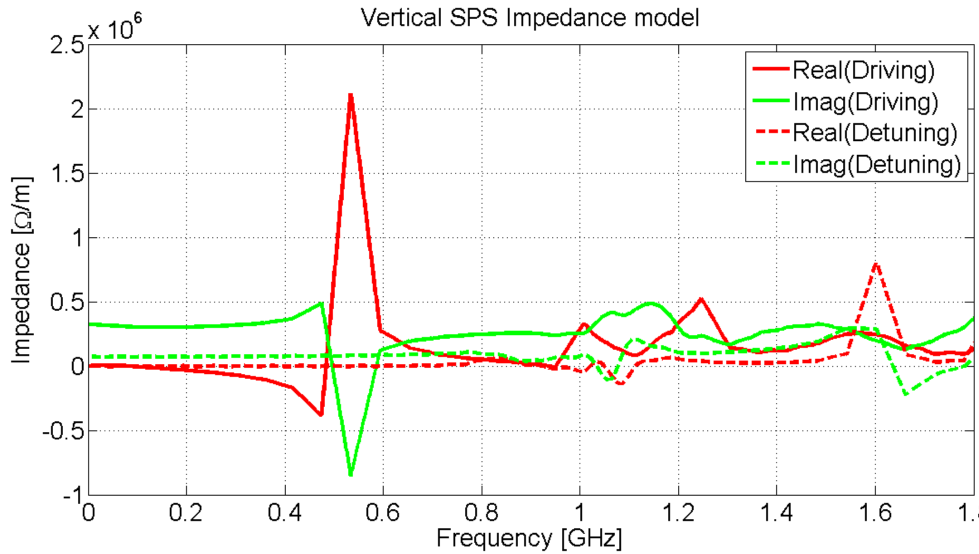
BPV





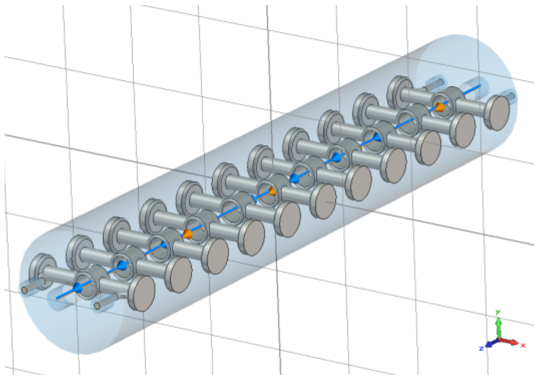


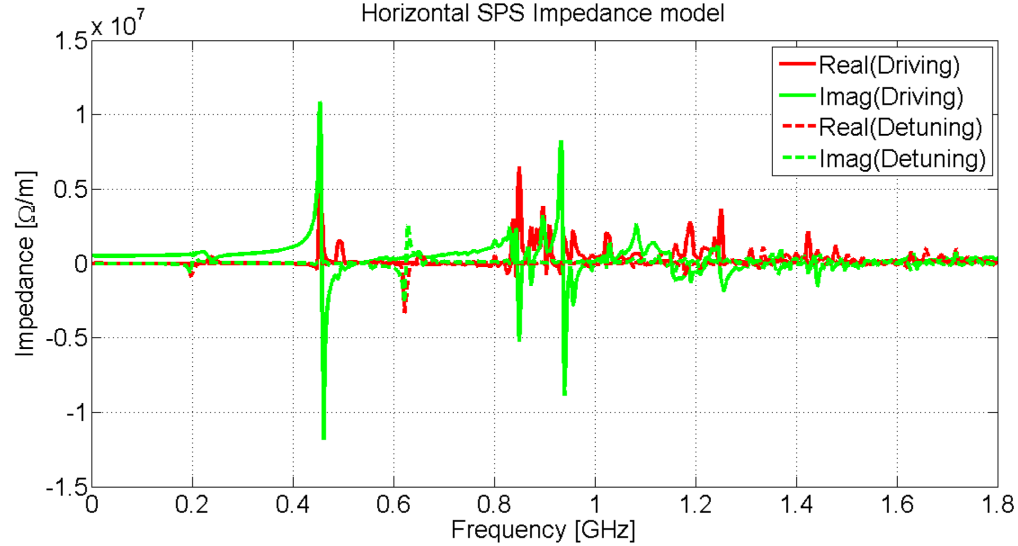


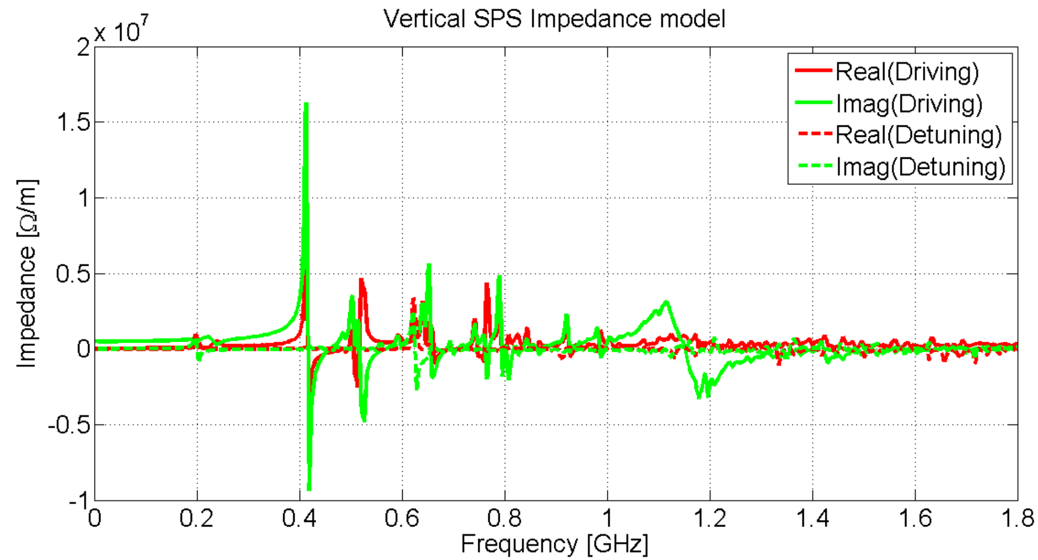


Impedance model of the SPS
The RF systems

200 and 800 MHz systems

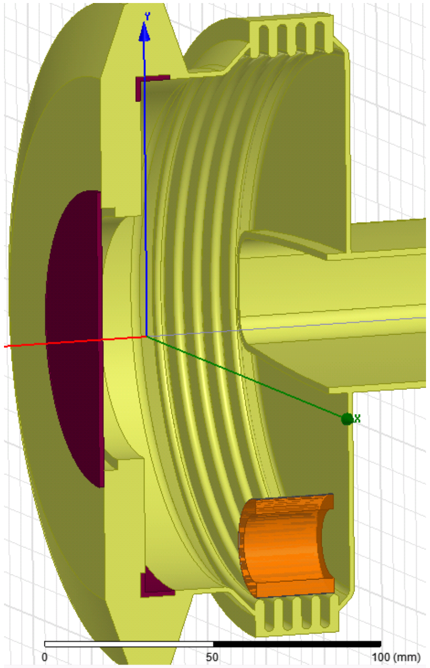






Impedance model of the SPS Transitions

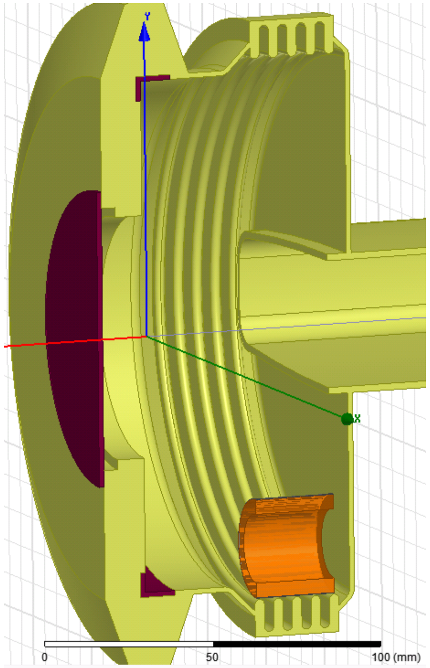
Step transitions between different types of chambers

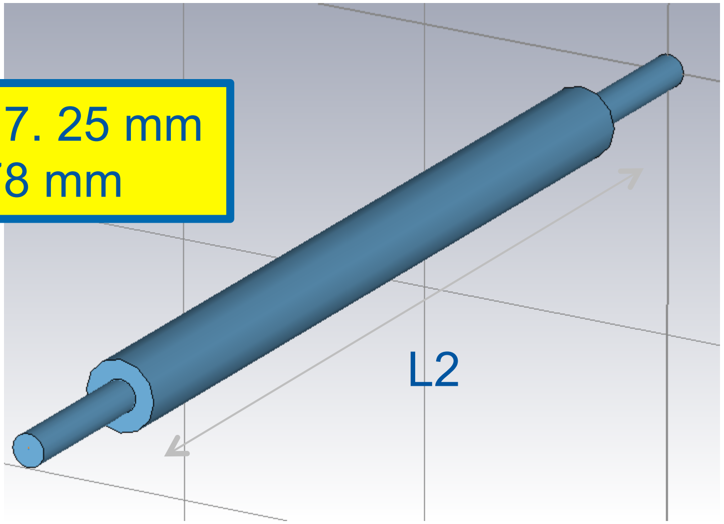


Flange Type	Enamel	Bellow	Num. of elements	Resistor
BPV-QD	Yes	Yes	90	No
BPH-QF	Yes	Yes	39	Long
QF-MBA	Yes	Yes	83	Short
MBA-MBA	Yes	Yes	14	Short
QF-QF	No	Yes	26	Short
QD-QD	Yes	No	99	No
QF-QF	No	No	20	No
BPH-QF	Yes	Yes	39	Long
QD-QD	No	No	75	No
QD-QD	Yes	No	99	No

Impedance model of the SPS
Transitions

Step transitions between different types of chambers





$$\mathrm{Re}[Z^{\mathrm{tr}}_{x,y}(\omega)] \approx 0$$

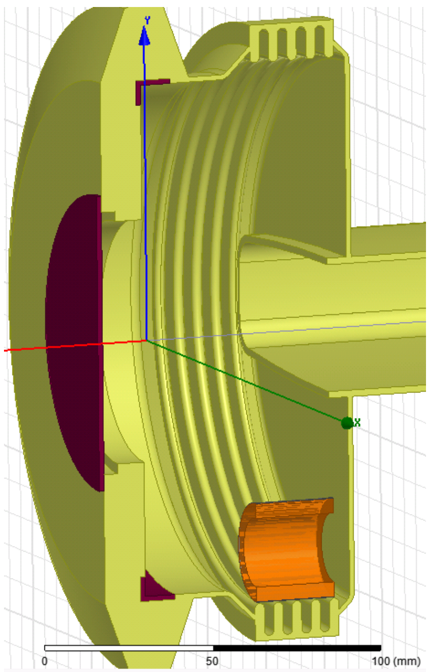
$$\mathrm{Im}[Z^{\mathrm{tr}}_{x,y}(\omega)] \approx 2\mathrm{Im}[Z^{\mathrm{step}}_{x,y}(\omega)]$$

since step-in and step-out have about zero real part of the transverse impedance and equal imaginary parts

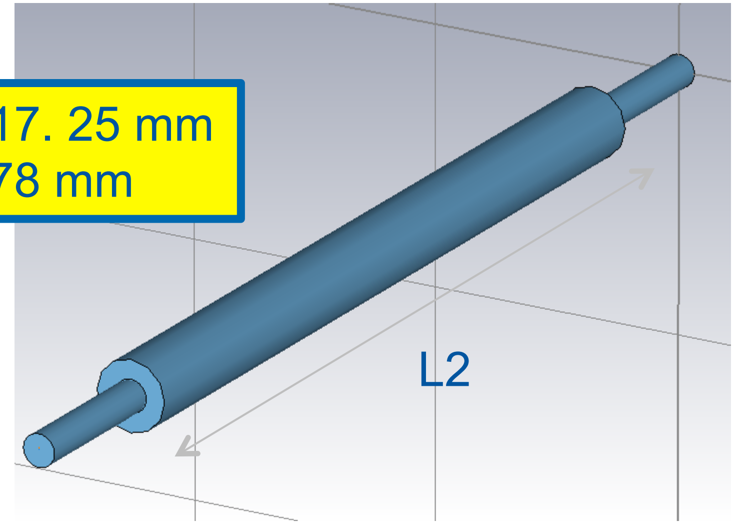
$$\mathrm{Im}[Z^{\mathrm{tr}}_{x,y}(\omega)] = \frac{1}{\langle\beta_{x,y}\rangle}\sum_{k=1}^{N^{\mathrm{tr}}}\mathrm{Im}[Z^{\mathrm{tr(k)}}_{x,y}(\omega)]n_k\beta^{\mathrm{tr}}_{x,y}$$

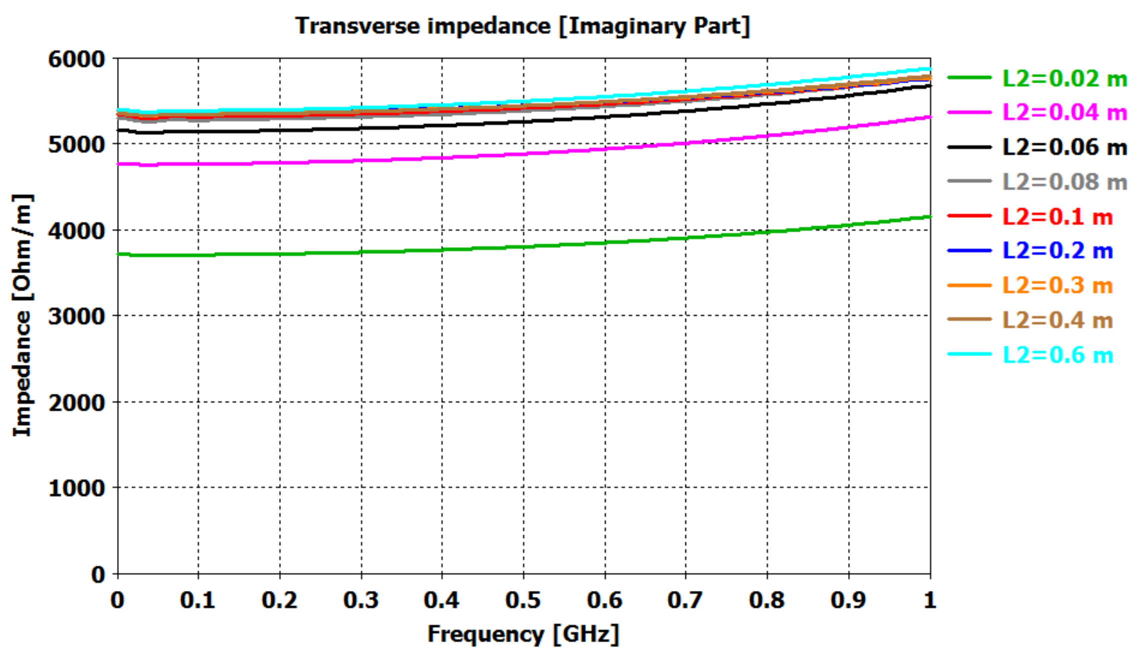
Impedance model of the SPS
Transitions

Step transitions between different types of chambers

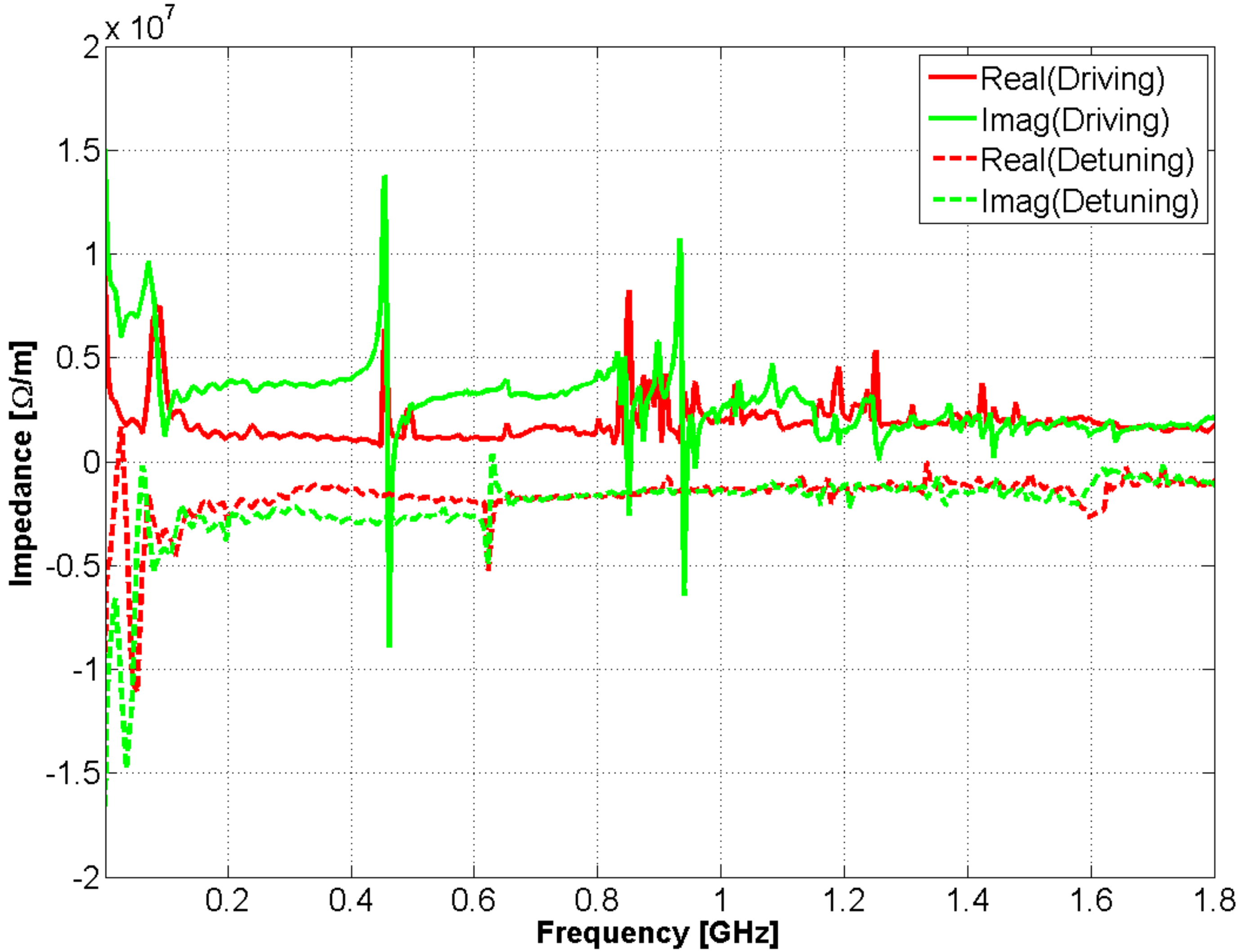


b = 17. 25 mm
d = 78 mm

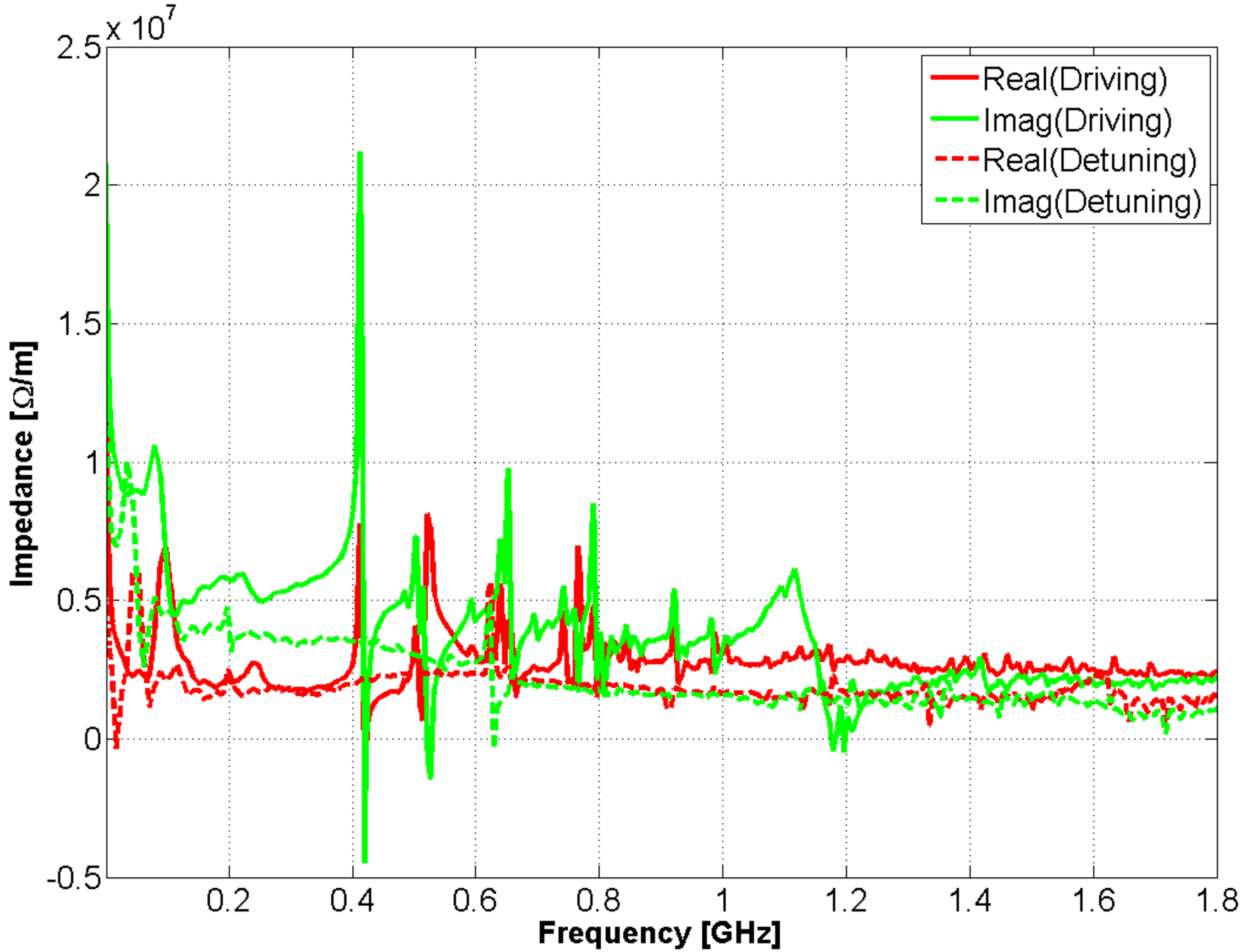




Impedance model of the SPS
Global horizontal impedance



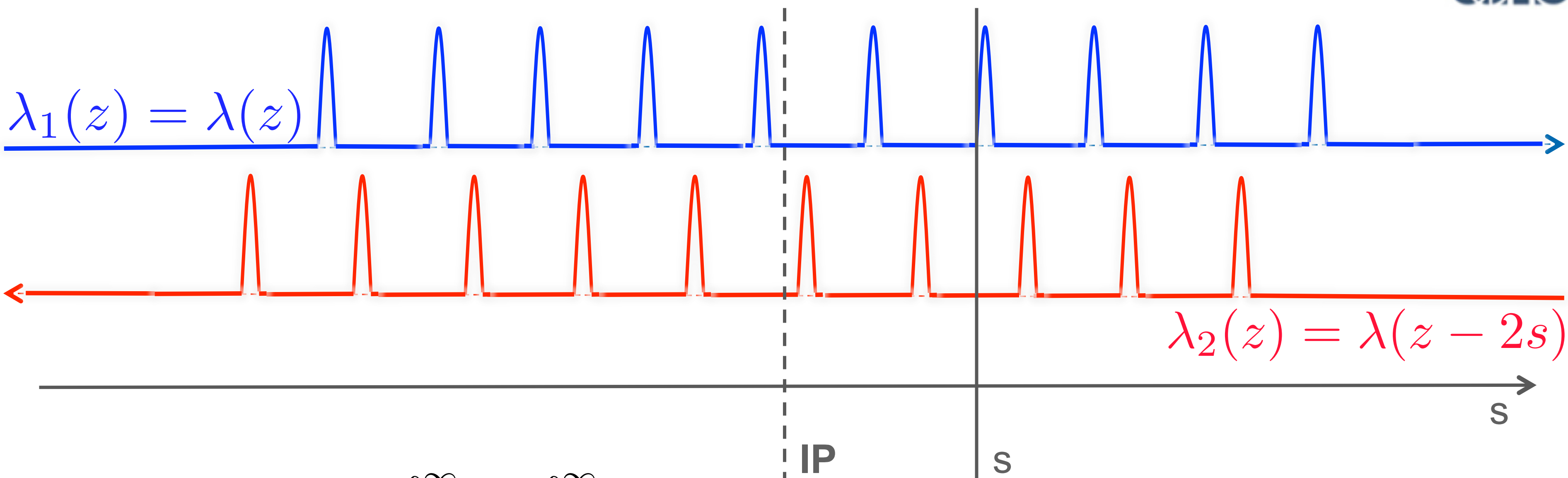
Impedance model of the SPS
Global vertical impedance



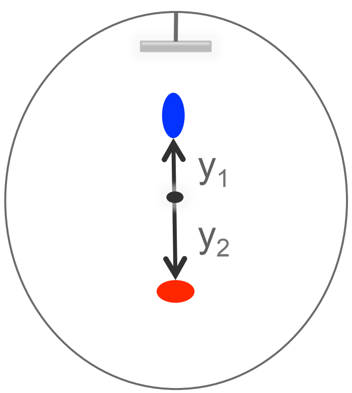
Summary

- Definition and properties of wake functions and impedances
  - Longitudinal wake and impedance
  - Transverse plane: dipolar and quadrupolar wakes and impedances
  - Panofsky-Wenzel theorem
- Examples of wakes and impedances
  - Resistive wall
  - Resonators
  - Complex structures
- Energy loss due to longitudinal wakes
  - General formulae
  - Some practical examples
- How to build the impedance model of a machine

A collider's asymmetric common chamber



$$\Delta E_{\text{beam1}}(s) = e^2 \int_{-\infty}^{\infty} \lambda(z) \int_{-\infty}^{\infty} \left[\lambda(z') W_{||b1}(z-z') - \lambda(z'-2s) W_{||b2}(z-z') \right] dz' dz$$

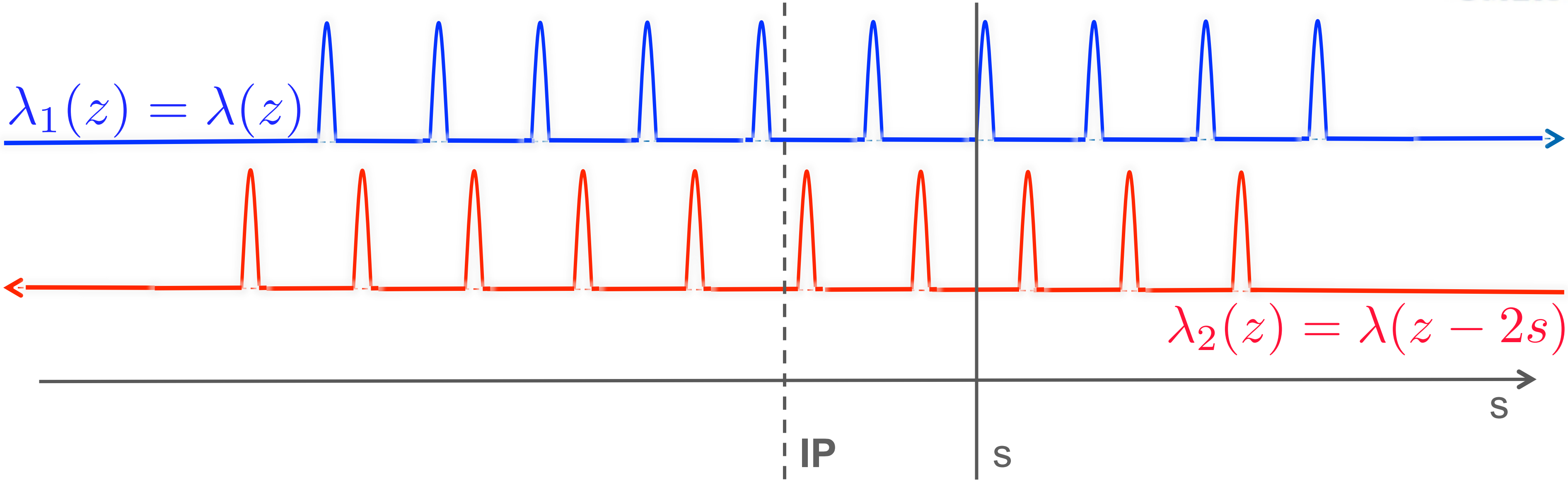


$$W_{||b1}(z) = W_{||}^{(0)}(z) + W_{||}^{(1d)}(z) y_1 + W_{||}^{(1q)}(z) y_1$$

$$W_{||b2}(z) = W_{||}^{(0)}(z) + W_{||}^{(1d)}(z) y_2 + W_{||}^{(1q)}(z) y_1$$

$$\text{with} \quad W_{||}^{1d}(z) = W_{||}^{1q}(z)$$

A collider's asymmetric common chamber



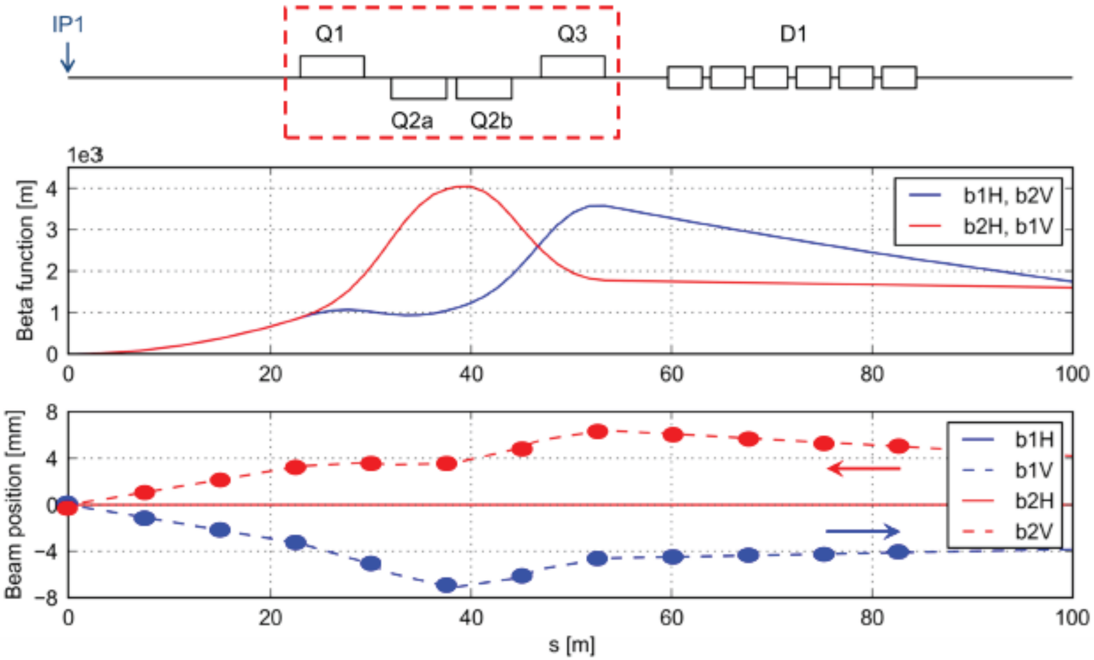
$$W_{||}^{1d}(z),\ W_{||}^{1q}(z) \overset{\mathcal{F}}{\Longleftrightarrow} Z_{||}^{1}(\omega) \qquad W_{||}^{0}(z) \overset{\mathcal{F}}{\Longleftrightarrow} Z_{||}^{0}(\omega)$$

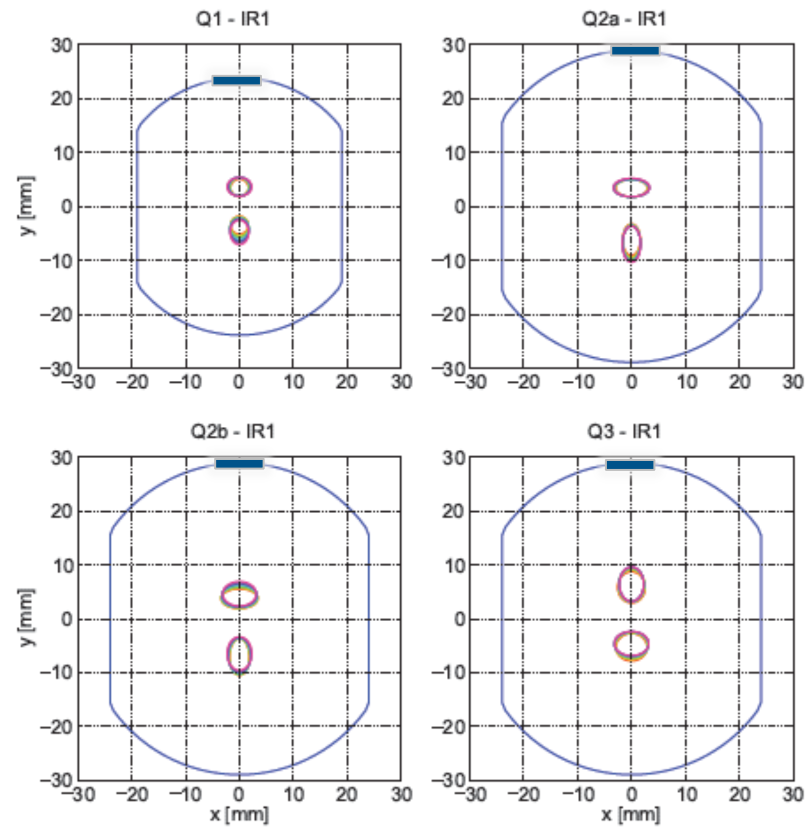
$$\Delta E_{\text{beam1}}(s) + \Delta E_{\text{beam2}}(s) =$$

$$\frac{4e^2\omega_0}{\pi}\sum_{p=0}^{\infty}|\Lambda(p\omega_0)|^2\left\{\text{Re}\left[Z_{||}^{0}(p\omega_0)\right] + [y_1(s)+y_2(s)]\,\text{Re}\left[Z_{||}^{1}(p\omega_0)\right]\right\}\cdot\sin^2\left(\frac{p\omega_0 s}{c}\right)$$

$$\Delta W_{CC} = \frac{\omega_0}{2\pi}\int_{-s_0}^{s_0}\left[\Delta E_{\text{beam1}}(s) + \Delta E_{\text{beam2}}(s)\right] ds$$

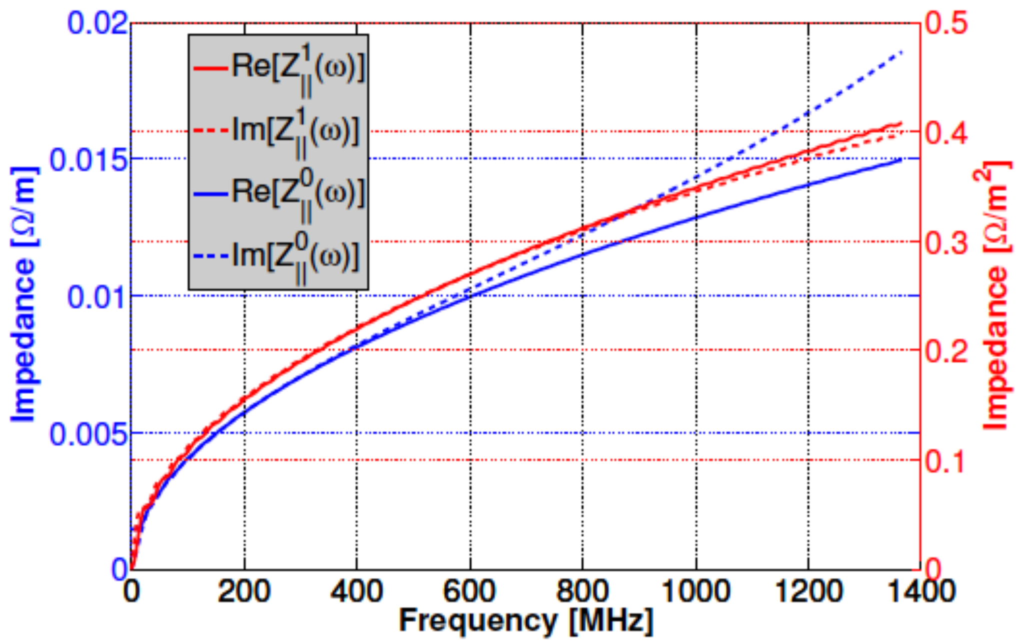
A collider's asymmetric common chamber





- Application to the LHC inner triplets
  - Beams are separated vertically (IP1) or horizontally (IP5)
  - Strongly off-axis for ~30m, all relative delays between beams swept
  - Asymmetric chamber in the direction of separation because of the weld

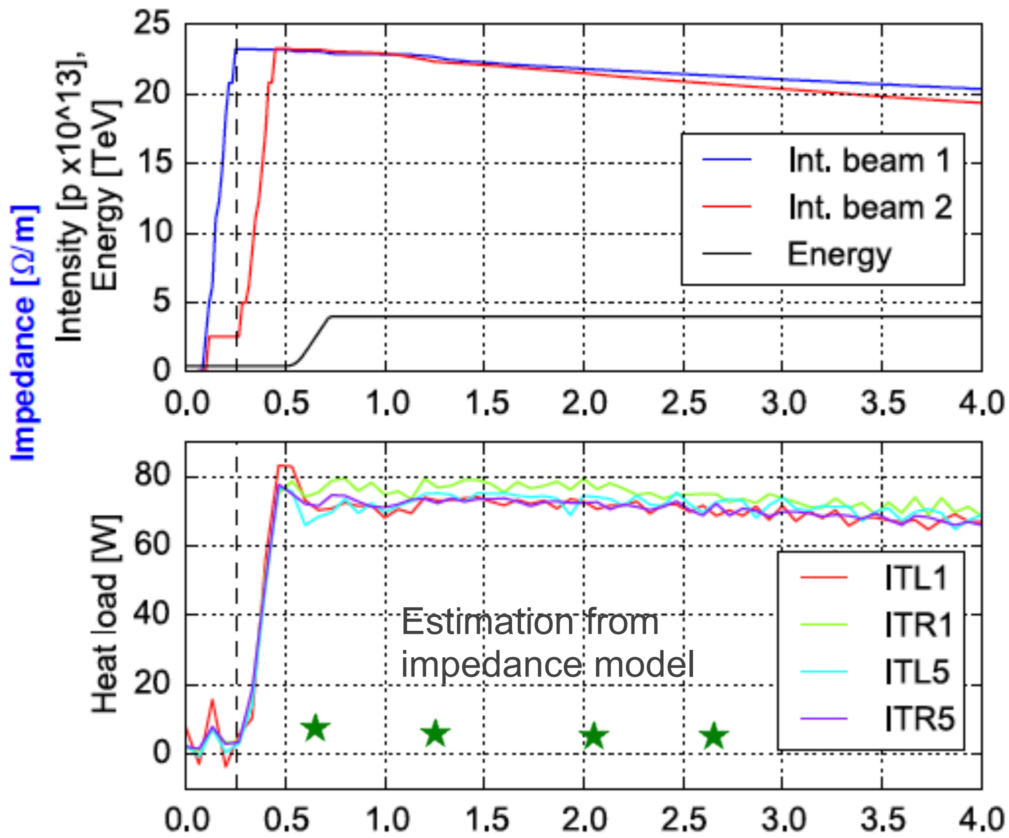
A collider's asymmetric common chamber



$$\Delta W_{IT} = 4\,\mathrm{W}$$

for a typical 50 ns fill of the LHC

A collider's asymmetric common chamber



- Comparison with measured data
  - Estimated heat load more than a factor 10 below measurement
  - Indication of a dominant contribution from something else (electron cloud), also enhanced by the two-beam effect

$$\Delta W_{IT} = 4\,\mathrm{W}$$

for a typical 50 ns fill of the LHC